Homeownership and Government-Provided Liquidity: Evidence from a California Lottery*

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Abstract

Most American renters would prefer to be homeowners and report that liquidity constraints prevent them from buying homes. We study a California state government program offering large down payment assistance loans to certain prospective first-time buyers. The loans charge a share of future home price appreciation and do not add to monthly costs. Leveraging conditional random assignment, we find that a loan offer more than doubles homeownership, and the effect persists a year after the deadline to use the loan. We estimate a model of non-financial preferences for homeownership, down payment capacity, and program participation costs that explains our reduced-form results. Participants value the loans primarily because they relax liquidity constraints that bind when meeting mortgage obligations, not because they subsidize housing consumption or help bypass minimum down payment requirements. Average willingness to pay for the California down payment assistance greatly exceeds expected fiscal cost, with a higher marginal value of public funds (MVPF) than housing voucher programs. Our MVPF estimate reflects high valuations among participants who would otherwise have bought with the smallest allowed down payment, and recovery of outlays through principal repayment and appreciation revenue.

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1 Introduction

Most American renters would prefer to be homeowners (Federal Reserve Board of Governors, 2025). The prevalence of this goal, its association with the proverbial American Dream, and a belief that homeowners make better citizens motivate an array of US policies promoting homeownership (Glaeser and Shapiro, 2003; Schwartz, 2021). Many policies specifically target homebuyers who use mortgage loans, for example, by enabling buyers to deduct mortgage interest from their taxes, and extending credit to buyers who can pay only a small fraction of a home's purchase price in cash. Despite these efforts, many aspiring homeowners report that they cannot afford to buy: when asked in a recent Federal Reserve survey why they did not own, 68% of renters explained they could not afford a down payment.

This paper studies how liquidity constraints affect the decision to buy a home and measures the potential for a novel financial product to improve welfare by relaxing them. The California Housing Finance Agency recently introduced a program, known as "Dream for All," which enabled participants to buy homes without any cash down payment. The program offered down payment assistance loans for up to 20% of a home's purchase price, or a maximum of $150K, to a limited number of prospective first-time, first-generation homebuyers. The loan has to be used with a 30-year conventional mortgage, but does not add to monthly mortgage payments; instead, it charges a share of future home price appreciation. Mortgage regulation makes it difficult for private financial institutions to offer loans on these terms, so the Dream for All program can be thought of as helping to fill a missing market.

Stratified random lottery allocation of down payment assistance loans in 2024 enables us to identify their causal effects. In the fourth quarter of 2025, one and a half years after the lottery, 70.9% of applicants who received a Dream for All offer had purchased homes, compared to 28.1% of applicants who did not. To the best of our knowledge, this reflects the largest documented effect of any intervention on homeownership in a high-income country. The effect is persistent, shrinking by less than five percentage points over the most recent four quarters for which data are available, consistent with Dream for All increasing, rather than merely retiming, homeownership.

We go on to analyze the mechanisms underlying this dramatic causal effect, considering the roles of two distinct borrowing constraints. Dream for All may have increased homeownership by exempting participants from the usual requirement that they pay a minimum share of the home's purchase price in cash in order to take out a mortgage. In particular, Dream for All-induced buyers may not have qualified for a mortgage without assistance because they were unable to make a minimum down payment. A second liquidity constraint that may deter prospective homebuyers, which Dream for All may also have relaxed, is the

requirement to meet consumption needs and pay mortgage obligations without drawing on future resources. Buying a home entails saving in an illiquid asset, and often takes place when consumption needs are high and earnings have not yet peaked.[1] Consequently, many young homeowners report being "house-poor," straining to meet consumption needs even as they accumulate home equity.[2] Dream for All may have increased homeownership by relaxing this homeownership-consumption tradeoff, reducing the consumption sacrifice associated with homeownership for program participants.

We develop a stylized model of housing tenure choice and financing to guide analysis of mechanisms. The model features both liquidity constraints introduced above: mortgage borrowing without assistance requires a minimum cash down payment, and unassisted homebuyers cannot borrow against future resources to help fund consumption in the period when they must make a down payment. Dream for All loans can substitute for cash, so they eliminate the minimum down payment and relax the homeownership-consumption tradeoff.

Under the model's assumptions, there are four types of applicants. Two are *compliers*, who prefer to buy with Dream for All assistance. *Marginal buyers* are compliers whose fallback option is renting and *inframarginal takers* buy unassisted if they cannot buy with Dream for All. Another two types of applicants are *never-takers*, unwilling or unable to use Dream for All assistance: *renter never-takers* and *buyer never-takers* rent and buy unassisted, respectively.

The model delivers sharp tests of whether and which liquidity constraints deterred homeownership based on the characteristics of the two complier groups. If Dream for All increased homeownership exclusively by enabling applicants to bypass the minimum down payment requirement for mortgage borrowing, every marginal buyer must be poorer than every inframarginal taker: inframarginal takers, by definition, can afford minimum down payments, and marginal buyers cannot. If Dream for All's effect on homeownership instead reflects relaxation of the homeownership-consumption tradeoff, marginal buyers and inframarginal takers may have similar resources. In this scenario, inframarginal takers with the same resources as marginal buyers strain to afford unassisted homeownership: when buying unassisted, they make low down payments and consequently borrow at high costs. We also consider the

[1] In particular, the median age of a first-time US homebuyer in 2024 was 38 years old (National Association of Realtors, 2024), over a decade before earnings reach their peak (Gourinchas and Parker, 2002). The age at which US households are most likely to have children at home, with associated high expenses, is 40 years old, according to authors' calculations from the 2024 American Community Survey one-year sample (Ruggles, Flood, Sobek, Backman, Cooper, Rivera Drew, Richards, Rodgers, Schroeder and Williams, 2025).

[2] The term "house-poor" is used in popular media, see e.g., Brooks (2024), but is consistent with scholarly research. Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru and Yao (2017) and Wong (2026) show that homeowners' consumption and financial distress are highly responsive to changes in housing costs, consistent with many homeowners being hand-to-mouth consumers. Boar, Gorea and Midrigan (2022) estimates that 80% of US homeowners are liquidity-constrained.

possibility that Dream for All did not alleviate any binding constraint, and the increase in homeownership merely reflects an appetite for subsidized housing consumption. In this case, Dream for All utilization must increase inframarginal takers' housing consumption.

We implement the model's mechanisms tests using the substitution framework of Kline and Walters (2016), which non-parametrically identifies the population shares and average characteristics of each of the model's four types of applicants. We find that 70% of compliers, who take up assistance when offered, are marginal buyers, and the remaining 30% are inframarginal takers. We reject the hypothesis that all marginal buyers are poorer than all inframarginal takers, implying that Dream for All's effect on homeownership did not entirely reflect minimum down payment elimination. The two groups' income distributions overlap substantially, and several other characteristics, including average credit scores, credit limits, and racial and ethnic composition are statistically indistinguishable across the two groups.

We find instead that Dream for All increased homeownership by relaxing a homeownership-consumption tradeoff. Key evidence comes from inframarginal takers' financing when buying unassisted. The modal inframarginal taker makes the minimum allowed down payment, and nearly three-quarters make down payments below 20%. These low down payments entail high future interest and mortgage insurance costs, which inframarginal takers tolerate because their present consumption is scarce when buying unassisted. Estimates of intensive-margin effects of Dream for All utilization bolster this interpretation: inframarginal takers used Dream for All loans not to buy more housing, but to reduce their down payment and mortgage borrowing. We extend this conclusion to marginal buyers by analogy: because marginal buyers resemble inframarginal takers on income and associated characteristics, we infer that unassisted homeownership would strain marginal buyers' consumption as it does inframarginal takers', and that this strain prevents them from buying unassisted.

This evidence informs calibration choices and enables us to estimate the joint distribution of non-financial preferences for homeownership, down payment capacity, and program participation costs in our model. We assume a simple parametric form for this distribution and estimate its parameters by classical minimum distance, targeting the estimated type shares and the distribution of unassisted down payments in the applicant population.

The resulting estimates enable us to measure willingness to pay for the Dream for All loan and compare it with a Dream for All loan's expected fiscal cost. Baseline estimates suggest that applicants' average willingness to pay for Dream for All is 7.1 cents per dollar of loan principal. The moment in the data that drives our willingness to pay estimate is the share of inframarginal takers making down payments at or below 5% of the purchase price, consistent with our argument that applicants primarily valued Dream for All for its relief from (non-housing) consumption strain. Taking the ratio of average willingness to pay with

the program's expected fiscal cost of 1.3 cents per dollar of principal gives a marginal value of public funds (MVPF) of 5.6. This is the first MVPF estimate for a US policy promoting homeownership. It is well above the benchmark value of 1 for a lump-sum transfer, and above MVPFs of 0.5–0.7 for US rental housing vouchers (Hendren and Sprung-Keyser, 2020; Ruby, 2024). To the extent that Dream for All recipients' incomes overlap with those of voucher recipients, this implies that Dream for All is a much more cost-effective way to help these households. Voucher recipients may be needier than Dream for All applicants, however, making them unsuitable as a comparison group. To compare Dream for All to a policy with the same target population, we evaluate the First-Time Homebuyer Credit using the model. Willingness to pay for the tax credit is nearly an order of magnitude higher than Dream for All assistance, but Dream for All's fiscal cost is nearly two orders of magnitude lower, so its MVPF is much higher. Simulations also suggest that Dream for All delivers more homeownership per dollar than the tax credit.

Contributions and related literature To the best of our knowledge, we are the first to calculate the MVPF for a policy promoting homeownership in the United States. The only estimate we are aware of for a homebuyer subsidy elsewhere, Ochoa (2025), also relies on a structural model of housing demand to recover households' valuation of homeownership. The abundance of US government policies promoting homeownership, from tax credits and tax deductibility of mortgage interest to government insurance for low down payment mortgages, makes it important to understand the cost-effectiveness of different policy levers (Glaeser and Shapiro, 2003; Poterba and Sinai, 2008; Glaeser, 2011).

This paper contributes directly to the literature on housing tenure choice. The paper's focal homeownership-consumption tradeoff is emphasized in early theoretical and structural scholarship (Artle and Varaiya, 1978; Slemrod, 1982; Henderson and Ioannides, 1983), but discussed little in more recent studies.[3] Structural studies of tenure choice largely focus on mortgage qualification, transaction costs, and taxes (Li and Yao, 2007; Díaz and Luengo-Prado, 2008; Chambers, Garriga and Schlagenhauf, 2009; Attanasio, Bottazzi, Low, Nesheim and Wakefield, 2012; Halket and Vasudev, 2014; Coven, Golder, Gupta and Ndiaye, 2024). Influential studies linking liquidity constraints to home price dynamics (see Piazzesi and Schneider, 2016, §3.7) prominently feature minimum down payments but tend to abstract

[3] In Henderson and Ioannides (1983), "renting is attractive since current consumption of all other goods is higher than under owning" (p. 110); the model does not feature a minimum down payment to qualify for a mortgage. Artle and Varaiya (1978) analyze a life-cycle model in which a household able to qualify for a mortgage may rent because the saving required for the down payment depresses early-life consumption, and a buyer exhausts her liquid assets at the moment of purchase. Slemrod (1982) presents a model in which down payments affect tenure choice not only through mortgage qualification, but because a down payment "disturb[s] the consumption stream" (p. 198).

from a homeownership-consumption tradeoff by assuming linear utility or removing tenure choice altogether (Stein, 1995; Ortalo-Magne and Rady, 2006; Favilukis, Ludvigson and Van Nieuwerburgh, 2017; Kaplan, Mitman and Violante, 2020).[4] Sommer and Sullivan's (2018) study of the mortgage interest deduction is an exception: many renters in their model could afford to buy but do not because the initial costs would "consume all of their savings and the majority of their one-period labor income" (p. 262). This paper provides experimental evidence that the tradeoff deters homeownership among households who can qualify for a mortgage, and uses a model in the spirit of Henderson and Ioannides (1983) to value a financial product which relaxes it. Future research can use the liquidity effect documented in this study as a moment to match in calibration or estimation.

Relatedly, a large empirical literature studies how liquidity and borrowing constraints affect homeownership. Some recent work uses quasi-experimental variation in liquidity generated by zero down payment mortgage availability from the US Veterans Administration (Fetter, 2013), hypothetical survey vignettes (Fuster and Zafar, 2016), exposure to the First-Time Homebuyer Credit (Berger, Turner and Zwick, 2020), student loan generosity (Goodman, Isen and Yannelis, 2021), and lottery jackpot size (Bulman, Goodman and Isen, 2023).[5] These quasi-experimental studies document a causal effect of liquidity on homeownership. Their effects are smaller than the effect we document, likely because the effect of liquidity on homebuying depends on intentions to buy a home: all Dream for All applicants in our study applied for financing to purchase a home, but not all lottery participants in Bulman, Goodman and Isen (2023), for example, may have planned to use winnings to buy homes. Because homebuying decisions in structural models are made by households weighing a purchase, we view our estimates as more suitable calibration targets than those of existing work.

Dream for All's null intensive-margin effects on housing consumption and mortgage borrowing are consistent with modest responses documented in the literature (DeFusco and Paciorek, 2017; Best, Cloyne, Ilzetzki and Kleven, 2020; Fuster and Zafar, 2021). Benetton, Bracke, Cocco and Garbarino (2022), in contrast, argue using a cross-region difference-in-differences analysis that a UK down payment assistance loan similar to Dream for All increased intensive-margin housing consumption. The contrast with our results may stem from the program's eligibility criteria: the Help to Buy program was not restricted to first-time homebuyers, and may therefore have been used by unconstrained buyers.[6]

[4] Kaplan, Mitman and Violante (2020) note that "taking advantage of looser PTI and LTV constraints requires sacrificing current consumption" (p. 3317) but do not quantify the importance of this tradeoff.

[5] Minimum down payments are especially salient liquidity-based lending standards, but not the only ones. In Greenwald (2018), maximum debt-to-income ratios deter homebuying; Bhutta and Ringo (2021) estimate their effects using variation in mortgage insurance premiums. We do not explicitly analyze these constraints, but they play a similar role as minimum down payments: they are barriers to qualifying for a mortgage.

[6] Another potential reason for our diverging findings is that interest rates were relatively low in Benetton,

Our study also relates to a literature on optimal mortgage design and alternative housing finance products (Campbell, 2013; Cocco, 2013). Caplin, Chan, Freeman and Tracy (1997) and Caplin, Carr, Pollock, Yi Tong, Tan and Thampy (2007) argue that shared equity mortgages can help relax the liquidity constraints discussed in this paper. Some recent studies measure the effects of alternative products using dynamic general equilibrium models (Greenwald, Landvoigt and Van Nieuwerburgh, 2021; Guren, Krishnamurthy and McQuade, 2021) and suggest that equity-type products can improve welfare. This paper gives the first experimental evidence on the effects of access to and utilization of such an alternative housing finance product, answering Shiller's (2014) call: "Alternative forms of mortgage need to be studied on a scientific basis. The New Development economics...have shown how carefully controlled experiments can reveal solid steps to take regarding new financial institutions for poverty reduction. The same methods could be used to improve mortgage institutions."

The remainder of the paper is organized as follows. Section 2 discusses the US mortgage market and the Dream for All program. Section 3 discusses the data we use for our main empirical analysis. Section 4 documents the causal effect of a Dream for All offer on homeownership. Section 5 introduces the stylized model which guides our analysis of mechanisms, and Section 6 carries out empirical tests of mechanisms. Section 7 parametrizes the model and conducts quantitative exercises. Section 8 concludes.

2 Setting

2.1 The US mortgage market

We briefly describe some relevant characteristics of the US mortgage market. Before starting to search for homes, prospective homebuyers meet with lenders who decide whether to extend credit, and on what terms, on the basis of credit profiles, income, and assets. The resulting mortgage pre-approvals specify a maximum loan amount. Pre-approval virtually always requires prospective borrowers to prove they can make a cash down payment.[7] The modal cash down payment is 20%, and borrowers must pay at least 3%.[8] Homebuyers who

Bracke, Cocco and Garbarino's study period (2013–2017), which may have made Help to Buy homebuyers more inclined than Dream for All borrowers to use additional liquidity to increase leverage.

[7]Two federal agencies make it possible for US veterans and low- and moderate-income buyers in rural areas to take out a mortgage without a cash down payment. These mortgages accounted for about 12% of home-purchase mortgage originations in 2023 (Consumer Financial Protection Bureau, 2024).

[8]Authors' calculations from 2023 HMDA data on first-lien, closed-end, owner-occupied, site-built one-to-four-unit home-purchase originations with a reported combined loan-to-value ratio: 15.4% of borrowers made a 20% down payment, compared with 12.1% at 3.5%, and 12.1% at 5%. A 3.5% down payment is the minimum to qualify for Federal Housing Administration (FHA) mortgage insurance, but the minimum to borrow with a government-sponsored enterprise (GSE)-backed mortgage is slightly lower, at 3%.

make small down payments incur higher borrowing costs than those who can make 20% down payments, due to higher interest rates, mortgage insurance premiums, and other fees. Mortgage pre-approvals are also contingent on other requirements related to liquidity, such as maximum debt-to-income ratios.

The most common mortgages in the US are 30-year fixed nominal interest rate loans which self-amortize, meaning that each payment covers some interest and some principal for a constant total payment over the loan's term. Both down payments and monthly payments therefore force borrowers to transfer their liquid resources into home equity.[9] Borrowers have little scope to borrow to help meet these mortgage obligations, so young first-time homebuyers often report that their mortgage obligations strain their budgets.

Savings accumulated as home equity are costly to extract: borrowers must either take out a second loan, in addition to their mortgage, or sell the home. Taking out a second loan (typically a home equity loan, home equity line of credit, or cash-out refinancing of the first mortgage) requires homeowners to have accumulated a minimum amount of home equity, pay closing costs, and to pass an underwriting process which involves verification of income, employment, and assets and a credit profile review.[10] Boar, Gorea and Midrigan (2022) estimate that a large share of US homeowners value a dollar of cash more than a dollar of home equity and would pay substantially to relax limits on borrowing against their homes.

Although state and federal governments rarely lend directly to homebuyers, government is widely regarded as setting the terms of the US mortgage market, and has historically been responsible for the introduction of new housing finance products.[11] A particularly important set of government regulations imposed through insurance and secondary market guidelines concerns the sources of down payment financing borrowers may use. The FHA, Fannie Mae, and Freddie Mac will insure or purchase mortgages only if any subordinate financing used toward the borrower's down payment comes from approved sources, such as governmental entities, nonprofits, or employers. These entities do not allow borrowers to receive down payment assistance from private for-profit lenders (Congressional Research

[9]Some borrowers who prefer to accumulate home equity more gradually take out interest-only (IO) mortgages, but these often carry higher interest rates (they are not insurable by the FHA and cannot be sold to GSEs), require higher down payments and proof of substantial cash reserves, and are usually available only for very large mortgages (the 2026 threshold was near $1.25 million in high-cost areas). We expect that almost no Dream for All applicants would have qualified for IO mortgages.

[10]These requirements may bind precisely when liquidity is most valuable: DeFusco and Mondragon (2020) show that income and employment verification prevented unemployed homeowners from refinancing, and Bhutta and Keys (2016) show that equity extraction is highly sensitive to mortgage interest rates, so that access to home equity dries up when rates rise.

[11]Schwartz (2021), for example, writes "The housing finance system has been structured to a large degree by the federal government, often in response to crisis. Many of the most enduring institutions and elements, including fixed-rate, self-amortizing mortgages, mortgage insurance, and a secondary mortgage market, stem from the Roosevelt administration's interventions in response to the Great Depression."

Service, 2024, see also HUD Handbook 4000.1). Restrictions on the source of down payment financing may help to explain the absence of a large private market for shared appreciation subordinate loans: a private entrant offering similar financing to Dream for All would need to pair its product with a mortgage ineligible for FHA insurance or GSE purchase. These non-standard mortgage products typically carry higher interest rates and less favorable terms than standard products.

2.2 The Dream for All Shared Appreciation Loan Program

The Dream for All Shared Appreciation Loan Program was established by the California Legislature in State Budget Trailer Bill AB 197 in 2022 (California Housing Finance Agency, 2025). The legislature charged CalHFA with administering the program, to which it originally allocated $500 million. In March of 2023 ("Phase 1"), CalHFA deployed $300 million. The 2023-24 State Budget (SB 104 of 2023) subsequently provided an additional $20 million to be used alongside the remaining $200 million to support a "Phase 2," which we study in this paper. The program aimed to "increase access to home ownership for low- and moderate-income Californians" (California Housing Finance Agency, 2025).

Dream for All loan terms Dream for All offered a subordinate loan for up to 20% of a home purchase, or a maximum of $150K, to use with a 30-year fixed-rate conventional mortgage. Borrowers may choose their loan size, but a 20% loan combined with an 80% first mortgage was the modal choice (Appendix Figure A.1). They could not make a cash down payment larger than 5% of the purchase price, and could use the Dream for All loan for closing costs, but not to cover closing costs beyond 5% of the purchase price.[12] Borrowers could buy single-family, one-unit residences, including manufactured homes and properties with an accessory dwelling unit, as well as condominiums. They had to intend to occupy properties as their primary residence within sixty days of closing. The shared appreciation loan carries no interest or amortization payments; instead, borrowers repay the principal plus a share of home price appreciation. If no appreciation occurs, borrowers still owe the full principal. Most Dream for All borrowers owe the financing share of appreciation (e.g., 20% of appreciation for a shared appreciation loan of 20% of the home's purchase price).[13] Borrowers earning below 80% of Area Median Income (AMI), as determined by Fannie Mae's HomeReady lookup tool, in the county where they purchased were given a "low-income"

[12]That is, the minimum combined loan-to-value (CLTV) ratio was 95%, and the maximum was 105%.

[13]In this sense, a Dream for All borrower writes a call option on the share of the home financed with the subordinate loan, struck at the purchase price, in exchange for interest-free financing. Equivalently, the borrower sells CalHFA that share of the home at the purchase price, writes CalHFA a put option on it struck at the purchase price, and occupies the home without paying rent on CalHFA's share.

appreciation rate of three quarters of the financing share (e.g., 15% of appreciation for a 20% investment by the state).[14] Appreciation costs are capped at 250% of the principal. Borrowers may not deduct the value of home improvements when calculating appreciation for repayment purposes.

Principal and the appreciation payment are due when the first mortgage is paid, so the maximum Dream for All loan duration is 30 years, as they must be used with 30-year mortgages. Other maturity events are home sale, (penalty-free) prepayment, cash-out refinancing, or default.[15] In the event of a home sale, the appreciation due to CalHFA is calculated based on sale price. If the loan is terminated without a sale, CalHFA reserves the right to choose an appraiser or use an automated valuation model to assess appreciation. If default occurs, CalHFA has the right to foreclose, but it is junior to the mortgage.

The financial advantage of buying with Dream for All over buying unassisted depends on how much cash a buyer has available and the return she earns on her liquid assets, as well as her home price expectations. To illustrate this, Appendix Table A.1 shows a buyer's net financial position seven years after purchasing a $500K home under two nominal appreciation scenarios, 50% and −10%, and three financing scenarios: a Dream for All-financed 20% down payment, a 20% cash down payment, and a 3.5% cash down payment. We assume the buyer has $100K available, earns 5% on her liquid assets compounded monthly, and takes out a fully-amortizing mortgage at 6.5% interest, plus mortgage insurance in the low down payment scenario. Buying with Dream for All results in a better net financial position than buying with a 3.5% cash down payment in both appreciation scenarios. However, buying with a 20% cash down payment is a better investment in the 50% appreciation scenario, while buying with Dream for All comes out ahead in the −10% appreciation scenario. Appendix Figure A.2 shows that at the median seven-year nominal appreciation of 26%, computed from repeat sales of California homes purchased in 1988 or later in the ATTOM recorder data and weighted by purchase price, Dream for All comes out narrowly ahead for a buyer earning 4.2%, the top high-yield savings rate as of October 2026 (NerdWallet, 2026), but buying with cash dominates for a buyer earning the FDIC national average savings rate of 0.4% (Federal Deposit Insurance Corporation, 2026).

Eligibility and allocation Dream for All applications could include multiple borrowers, but all had to be "first-time homebuyers," which, for program purposes, meant not owning

[14] CalHFA defines the low-income tier as credit-qualifying income at or below 80% of AMI as determined by Fannie Mae's HomeReady® Area Median Income Lookup Tool (California Housing Finance Agency, 2024). Fannie Mae's AMI estimates are FHFA's area-level median family incomes, which are derived from HUD's median family income estimates.

[15] Borrowers are allowed one rate-and-term refinancing of their first mortgage without triggering repayment.

their home for the past three years. At least one applicant per application had to be a "first-generation" homebuyer, meaning that their parents do not own a home in the United States and the applicant herself had not owned her own home for the seven years preceding her application. At least one applicant per application had to be a California resident. Applicants also had to earn total credit qualifying income below county-specific limits corresponding to 120% of AMI. Figure 1 Panel A shows these limits, which ranged from $132K in rural Humboldt County to $287K in Silicon Valley's Santa Clara County, although fewer than 3% of applications reported credit qualifying income above $200K. Borrowers did not have to buy in the county from which they applied, but had to earn below income limits in the purchase county. For example, a Dream for All recipient in Santa Clara County earning $150K would not be allowed to buy with Dream for All in Humboldt County, where she exceeds the $132K limit, but she could buy in Alameda County, where the limit was $246K.

Applicants had to complete a homebuyer education course and receive mortgage and subordinate loan pre-approval from a CalHFA-approved lender; these are widely distributed through the state and include some national lenders. As usual, pre-approval was contingent on credit scores, income, and assets; however, Dream for All pre-approval did not depend on ability to make a cash down payment. The minimum credit score was 660. Pre-approval required borrowers to give proof of income, such as pay stubs or tax returns, and assure lenders that they would continue to earn a similar income after buying a home.[16]

Dream for All Phase 2 allocated assistance by a random lottery process stratified by regions (county groups) shown in Figure 1 Panel B.[17] Applicants registered in April 2024 by submitting demographic and eligibility information, and uploading a mortgage pre-approval letter from an approved lender, via an online portal. Over 18,000 applicants submitted complete applications and were notified in June whether they were offered a Dream for All loan, placed on a waitlist, or not selected; 1,700 were initially offered loans (California Housing Finance Agency, 2025). Dream for All loan offers expired after 180 days, after which the offer was forfeited to an applicant on the waitlist.[18] Panel C of Figure 1 shows the distribution of lottery outcomes across regions. When a Dream for All borrower entered into a contract with a home seller, CalHFA undertook a final eligibility verification. The lender from whom the applicants obtained pre-approval then originates both the Dream for All loan and the first mortgage, which were purchased by CalHFA, assuming re-verification

[16]This may have limited mobility across county borders, and limits the scope for Dream for All utilization to have exerted a causal effect on earnings.

[17]The number of initial and waitlist offers was determined so that each region was expected to receive program funding in proportion to its share of the California population.

[18]Some extensions were granted, in particular, to applicants residing in the Los Angeles region due to the wildfires that occurred in late 2024.

passed. CalHFA then sold the mortgage to Fannie Mae.

3 Data

This section describes the data we use in our analysis. Our data comprises the California Housing Finance Agency's administrative records, the University of California Consumer Credit Panel (UCCCP), the ATTOM Recorder commercial database, the Landvision assessments database, and a handful of auxiliary sources. After describing these data sources, we present descriptive statistics and test for balance among the treatment and control groups in our analysis sample.

3.1 Data sources

California Housing Finance Agency administrative records We identify subjects for our study and track their Dream for All loan utilization using the California Housing Finance Agency's administrative application and loan utilization records for 2024 applicants. The application data gives demographic information (age, race, ethnicity, household size, and gender) for every co-borrower listed on an application. One co-borrower on every application is designated the "primary borrower." The data also gives eligibility-related information, such as the total credit-qualifying income among all borrowers, and loan preapproval amounts for the first mortgage and Dream for All loan. Loan utilization records give the actual first mortgage and subordinate loan amounts used, the type of property purchased (e.g., single-family home, condo, ADU), the date each loan was used, and the property's location. Loan utilization records run through June of 2025.

University of California Consumer Credit Panel The UCCCP is a credit bureau database which covers the consumer credit activity of everyone who has lived in California from 2004 to the fourth quarter of 2025. The data are updated quarterly, and are de-identified to protect borrowers' privacy. We use the UCCCP to primarily observe whether a Dream for All applicant purchased a home using a mortgage, in what geographic area the home purchase occurred, and on what mortgage terms. We merge CalHFA and UCCCP data on hashed names, dates of birth, and address following the algorithm developed by Fu, Davis, Rothstein, Ramesh and White (2022), achieving a merge rate of 72%. Appendix B discusses the merge.

Property and transaction data We use ATTOM and Landvision commercial data sets to observe property and transaction characteristics. Landvision data include property char-

acteristics such as building square footage and number of bedrooms and bathrooms. We use the ATTOM data to measure down payments and house acquisition price. We merge both these data sources onto UCCCP data using mortgage loan amounts, mortgage date, and mortgage borrower ZIP codes, successfully merging 74.4% of mortgages taken out by Dream for All applicants to ATTOM records and 52.3% to Landvision records.

Neighborhood data We obtain data on the characteristics of Census tracts, ZIP codes, and counties in California from various sources to characterize the origin and destination neighborhoods of Dream for All applicants. We use data on local labor market characteristics – such as wage growth, job growth, job density, employment rates, and commute times – and local neighborhood quality – including average family income for children with parents at the median of the income distribution, poverty rate, college attainment, test scores, teen birth, and single parenthood – from the Opportunity Atlas, as introduced in Chetty, Friedman, Hendren, Jones and Porter (2026). We follow Golosov, Graber, Mogstad and Novgorodsky (2024) in additionally measuring neighborhood quality using the Childhood Opportunity Index (Noelke, McArdle, Baek, Huntington, Huber, Hardy and Acevedo-Garcia, 2020) and Area Deprivation Index (Kind and Buckingham, 2018; University of Wisconsin School of Medicine and Public Health, 2024). Finally, we measure the owner-occupied share of housing at the Census tract level using American Community Survey data.

3.2 Analysis sample definition and characteristics

We use a subset of the CalHFA-UCCCP linked sample which was either initially offered a Dream for All loan (treatment group) or not selected (control group).[19] Our analysis sample comprises one applicant per application: we select the applicant who indicated on the application that they would be the primary borrower for the Dream for All-financed home purchase if they received a Dream for All offer. Finally, we exclude applicants from the "Rural Areas" region because all applicants in that region were either offered a loan or waitlisted (Figure 1 Panel C).

Figure 2 shows covariate balance across treatment and control groups and highlights key features of the analysis sample; Appendix Table A.2 shows additional summary statistics.[20]

[19]We exclude waitlisted applicants to avoid complications stemming from anticipation of possible treatment, and mechanical bias from correlation between initially treated applicants' complier status and waitlisted applicants' selection probabilities (van Dijk, 2019; De Chaisemartin and Behaghel, 2020).

[20]The summary statistics shown in Figure 2 are averages across regions with equal weight, while those reported in Appendix Table A.2 are raw means across all applicants; the two differ most for the Hispanic or Latino share (35% vs. 29%). The average age, income, and mortgage pre-approval quoted in the text are the raw means from Table A.2.

The modal Dream for All applicant is aged between 30 and 40, and the average applicant age is 40.5 years old. Less than half of primary borrowers are female. The average household size reported on applications is 3; although we cannot directly infer the ages of non-applicant household members, this suggests that the typical applicant household had at least one child at home. Less than half of applicants reported that their race was white, which we conjecture was related to the requirement that at least one borrower per application was a first-generation homebuyer. Hispanic or Latino applicants make up 35% of the sample. The modal applicant had credit qualifying income between $100K and $150K; the average income in the sample was $121K. The modal mortgage pre-approval contingent on Dream for All loan receipt was between $300K and $600K; the average was $511K. Applicants' credit scores were well above the program minimum of 660: over four-fifths had credit scores above 700, and more than half had credit scores above 750. On average, applicants had 0.5 open auto loans, 4.3 open credit cards, and 1.1 open student loans. Most had recent borrowing activity – 86% had at least one inquiry since 2022 – but only 1% were late on any payments in the quarter of application, and only 2% had any public record (generally bankruptcy). An F-test for balance across variables shown in Figure 2 yields a p-value of 0.87.

4 Causal effect on homeownership

Figure 3 shows the causal effect of access to a Dream for All loan on homeownership. Panel A shows that 70.9% of the treatment group had an open mortgage six quarters after the Dream for All lottery occurred, compared to 28.1% of the control group.[21] This implies a 42.8 percentage point causal effect on homeownership, or a relative increase of over 150%; in regression-adjusted terms (Panel C), the effect shrinks slightly to 42.5. To the best of our knowledge, this is the largest documented causal effect of any intervention on homeownership in a high-income country.[22] The causal effect is larger than the cross-sectional 25 percentage point homeownership gap between households in the second quartile and top 10% of the wealth distribution according to the 2022 Survey of Consumer Finances, among whom median wealth is $93.4K and $3.8M, respectively. It is also larger than the 30 percentage point Black-white gap in homeownership (U.S. Census Bureau, 2026); according to the Survey of Consumer Finances, median wealth of these two groups is $44.1K vs. $284.3K,

[21] We use an open mortgage as a proxy for homeownership, which we believe is a reasonable choice in this setting because cash purchases of homes are unusual among first-time homebuyers. They would be particularly unusual for applicants who undertook the hassle of the Dream for All application process.

[22] The second largest documented effect is Fuster and Zafar's (2016) reported effect of 40.2 percentage point effect of a hypothetical $100K inheritance on intentions to own among renters ($N = 319$) in a survey experiment. The second-largest real-world intervention is documented in Bulman, Goodman and Isen (2023), discussed below.

respectively. For reference, the modal Dream for All subordinate loan pre-approval was for $150K, and the average pre-approval was for $121K (Figure 2, Appendix Table A.2): the causal impact of a relatively modest change in liquidity increased homeownership more than large cross-sectional differences in wealth. The Dream for All effect is also larger than the 37 percentage point effect of winning one million dollars in the lottery, relative to a $5,000 win, documented by Bulman, Goodman and Isen (2023). This suggests that preparation and intent to buy are important mediators for the effect of liquidity on homebuying: all Dream for All applicants were prepared to buy a home, assuming they received assistance, while many renters who won the lottery may not have been prepared to buy or interested in buying.

Panels B and C of Figure 3 show that the Dream for All effect on homeownership was persistent. Panel B shows the share of the treatment and control group with open mortgages from the third quarter of 2023 to the fourth quarter of 2025, and Panel C shows the corresponding causal effect of a Dream for All offer at each time horizon, measured as the coefficient β from the regression

$$\text{BUY}_i = \alpha + \beta \text{LOTTERY}_i + \mathbf{R}_i \Gamma + \varepsilon_i$$

where BUY_i is an indicator for having an open mortgage at a given time horizon, LOTTERY_i is an indicator for receiving a Dream for All lottery offer, and $\mathbf{R}_i$ is a vector of randomization strata. No applicants had open mortgages before the lottery took place in the second quarter of 2024, reflecting the program's first-time, first-generation homebuyer eligibility requirement.[23] Homebuying among both groups rose slightly in the quarter following the lottery, but the treated group diverges sharply in the fourth quarter of 2024. The regression estimates suggest that Dream for All caused a 47.1 percentage point increase in homeownership four quarters after the lottery offers, which shrinks to a 42.5 percentage point effect at six quarters as control group applicants continue to buy homes. The proportionally modest reduction in treatment effect size addresses a natural concern that the dramatic increase in homebuying in the first few quarters reflected retiming of home purchases to take advantage of a time-limited subsidy. The results are consistent with Dream for All increasing, rather than merely retiming, homeownership for the overwhelming majority of applicants.

These dramatic effects raise two questions. First, why did applicants prefer Dream for All to status quo financing so much that over twice as many applicants bought homes when they had access to assistance than when they did not? Second, do applicants' valuations

[23]While atypical, a first-time homebuyer according to program guidelines could have a mortgage against a property they did not occupy as their primary residence. This appears not to have been the case in our analytic sample.

of the Dream for All loan justify program expenditures? The following section introduces a simple model to help address these questions.

5 A model of housing tenure and financing

This section models the choice of whether to rent or buy a home and, if buying, how to finance the purchase.[24] There is no uncollateralized credit; agents can only borrow against a home, and mortgage borrowing requires a minimum down payment. No state-contingent contracts are available to insure against home price risk.[25] This section assumes away mortgage default, which we consider in the quantitative analysis of Section 7.

5.1 Assumptions

Agents live for two periods $t = 0, 1$. A shared appreciation loan (SAL) down payment assistance program is announced at $t = 0$. A unit mass of agents, a small subset of the economy, are eligible according to exogenous criteria. Each eligible agent i is endowed with wealth $w_i > 0$ and a (non-financial) preference for homeownership over renting $\phi_i > 0$, which may represent, for example, utility from control of the housing asset or access to better-maintained housing. Using an SAL entails a hassle cost κ which varies across eligible agents; we assume $\kappa_i \in \{0, \infty\}$. Infinite hassle cost can represent inability to comply with SAL guidelines (time limits, property eligibility requirements) and/or discrimination by sellers against buyers with non-standard financing (Gilbukh, Conklin, Lopez and Ambrose, 2024). Wealth and the homeownership preference are jointly continuously distributed across the eligible population according to a cumulative distribution function (CDF) F. The probability of infinite hassle cost may depend on wealth and preference for homeownership.

Eligible agents receive exogenous income in period 1 and save in government bonds, which return net rate ρ. They decide in period 0 whether to buy or rent a home, as well as their non-housing and housing consumption levels. We denote per-period non-housing consumption c_0, c_1 and housing consumption h.[26] Agents must fund initial consumption c_0

[24] The model's closest antecedent is the two-period housing tenure choice model of Henderson and Ioannides (1983), which also features home price risk and assumes away uncollateralized credit. We add a mortgage rate schedule depending on the down payment size (as in Kaplan, Mitman and Violante, 2020), and consider a second lien whose repayment is contingent on home sale price.

[25] These restrictions mirror the actual mortgage market. Various primitive market failures may explain the missing markets in our model, including asymmetric information, moral hazard in maintenance and sale, or regulatory barriers to entry. Our analysis takes as given the menu of available contracts.

[26] We may conceive of h as an index of a home's characteristics, including both property characteristics and neighborhood amenities, provided that utility, purchase price, rent, and resale price all depend on the characteristics only through that index; the composition of the bundle is then indeterminate, and Section 6

and any cash down payment out of their wealth endowments. In period 1, they liquidate their assets, repay debts, pay rent (if they choose not to buy), and consume net resources.

Buying or renting a home Agents may buy housing in period 0 at price p_0 per unit h using cash, mortgage, and/or SAL financing. Homebuyers sell in period 1 at stochastic per-unit price p_1. Period 1 is a high home price state ("state H") with probability π, in which case $p_1 = p_H$; otherwise, in "state L," $p_1 = p_L$. We assume $p_H > p_0 > p_L > 0$.

Homebuyers choose which shares of a purchase to finance with cash, SAL, and mortgage, denoted θ_{cash}, θ_{sal}, and $1 - \theta$, respectively, with $\theta \equiv \theta_{\text{cash}} + \theta_{\text{sal}}$. Mortgage financing cannot fund an entire purchase: the maximum allowed mortgage-to-value ratio is $1 - \underline{\theta}$.[27] The mortgage interest rate depends on the mortgage financing share $1 - \theta$ according to a rate schedule function $r(1 - \theta)$, assumed nonnegative, twice differentiable, strictly increasing ($r'(\cdot) > 0$), and convex ($r''(\cdot) \geq 0$) in $1 - \theta$.[28] We assume the rate on a maximally mortgage-financed purchase is no lower than the return on household savings ($r(1 - \underline{\theta}) \geq \rho$) and the rate on an infinitesimally mortgage-financed purchase is strictly lower ($r(0) < \rho$).[29]

SAL financing requires repayment of the principal $\theta_{\text{sal}} p_0 h$ plus a share $\gamma \cdot \theta_{\text{sal}}$ of (positive) home price appreciation. The appreciation-sharing parameter γ determines the ratio between the share of appreciation owed in period 1 and the SAL financing share θ_{sal}. Recall that the "normal" rate for Dream for All sets $\gamma = 1$, while the low-income rate sets $\gamma = 0.75$.

The *user cost of owner-occupied housing* is the lifetime cost of buying a unit of housing: any down payment plus the present value of financing costs minus home sale proceeds,

$$\theta_{\text{cash}} p_0 + \frac{(1 + r(1 - \theta))(1 - \theta) p_0 + \theta_{\text{sal}} p_0 + \gamma \theta_{\text{sal}} (p_1 - p_0)^+ - p_1}{1 + \rho}.$$

Starting from the maximum loan-to-value ratio $1 - \underline{\theta}$, the user cost declines as the loan-to-value ratio falls towards the cost-minimizing down payment share characterized below, so agents who can afford larger down payments face a lower user cost.

Agents may alternatively rent at per-unit cost $q > 0$, paid in period 1. We assume

measures its components separately.

[27] In practice, minimum down payments are set by mortgage insurers and the government-sponsored enterprises: FHA requires a minimum homebuyer investment of 3.5 percent of the purchase price, and conforming loans require at least 3 percent down. Approved down payment assistance counts towards these requirements, consistent with our assumption that SAL financing satisfies $\underline{\theta}$. Theoretical models usually rationalize minimum down payments as lender protection against default (see e.g., Campbell and Cocco, 2015); this section assumes default away and imposes $\underline{\theta}$ directly.

[28] Typically default risk is the justification for this mortgage rate schedule shape. In our simple model, which abstracts from default risk, the assumption can be justified by convex mortgage origination costs.

[29] This makes the first dollar of down payment beyond the minimum attractive relative to a dollar of saving, and the first dollar of mortgage financing attractive relative to a marginal dollar of down payment, respectively.

rent is no higher than the minimum expected user cost of owner-occupied housing, that is, the lowest expected user cost attainable with any combination of cash, mortgage, and SAL financing.[30] Precisely, the assumption is

$$\frac{q}{1+\rho} \leq \min_{\substack{\theta_{\text{cash}} \geq 0,\ \theta_{\text{sal}} \geq 0 \\ \theta_{\text{cash}} + \theta_{\text{sal}} \in [\underline{\theta}, 1]}} E\left[\theta_{\text{cash}} p_0 + \frac{(1 + r(1-\theta))(1-\theta)p_0 + \theta_{\text{sal}} p_0 + \gamma \theta_{\text{sal}} (p_1 - p_0)^+ - p_1}{1+\rho}\right]. \tag{A1}$$

Agents who cannot make a cost-minimizing down payment face a strictly higher user cost of ownership than renting.

Government The government offers SALs on the terms described above, and values SAL cash flows at their expected value because it can spread realized gains and losses thinly over the full tax base.[31] Its cost of capital is the net rate it pays on its bonds, ρ.

Assuming mortgages are provided by absentee financiers whose welfare is immaterial, the government can generate a Pareto improvement by offering an actuarially fair SAL. In the Appendix C (Proposition 1), we show that an SAL is Pareto-improving if it has positive take-up and its appreciation-sharing parameter weakly exceeds γ_{fair}, defined as

$$\gamma_{\text{fair}} = \frac{\rho}{\pi g^+}$$

where ρ is the government's cost of capital, π is the probability of appreciation, and $g^+ \equiv \frac{p_H - p_0}{p_0}$ is the rate of home price appreciation conditional on appreciation occurring. Intuitively, if the government is at least compensated for their cost of capital in expectation, they provide a product without any expected cost which is valued by the eligible population. If $\gamma_{\text{fair}} \leq 1$, Dream for All represented a Pareto improvement. Section 7 conducts a quantitative evaluation in which Dream for All can generate welfare gains even if $\gamma_{\text{fair}} > 1$, that is, housing upside fails to compensate the government for its cost of capital. We also relax the assumptions that the government is risk-neutral and agents always pay their debts.

[30] This assumption is consistent with rental housing being supplied by risk-neutral, competitive landlords with access to the same financing as households. In practice, landlords do not have access to the SAL analyzed in this paper but may use equity financing which offers similar benefits.

[31] Lucas and Phaup (2010) and Lucas (2012) argue that, contrary to this assumption, taxpayers who ultimately bear risks should value cash flows at the rate the market applies to cash flows of equivalent risk. Novy-Marx and Rauh (2009, 2011) argue this point specifically for state pension liabilities. Home price appreciation is procyclical, so the state's SAL claim is worth less to taxpayers than its expected value discounted at ρ. Section 7 relaxes the assumption of government risk neutrality.

Preferences Agents have identical Stone-Geary preferences over housing and non-housing consumption of the form

$$U(c_0, c_1, h) = \beta \ln(h - \underline{h}) + (1 - \beta) \sum_{t=0}^{1} \frac{1}{(1+\rho)^t} \ln c_t$$

Period-0 consumption is the numeraire. Agents discount at their savings rate ρ. The preference weight $\beta \in (0, 1)$ determines marginal propensities to spend on housing and non-housing consumption out of disposable income. We assume period-1 income covers rent on the subsistence housing bundle, and covers the period-1 shortfall on a subsistence house whose full price is repaid at the maximum mortgage rate if it sells below its purchase price,

$$y > q\underline{h}, \qquad y > [(1 + r(1 - \underline{\theta}))p_0 - p_L]\underline{h}. \tag{A2}$$

Preference for homeownership ϕ and hassle cost of SAL utilization κ are additively separable from preferences for housing and non-housing consumption. Indirect utilities from renting, unassisted ownership, and ownership with the SAL are, respectively,

$$\begin{aligned}
V_R &= \max_{c_0, h} E\left[U(c_0, c_1, h)\right] \quad \text{s.t.} \quad c_0 \leq w, \quad c_0 + \frac{c_1}{1+\rho} = w + \frac{y - qh}{1+\rho}, \\
V_O &= \max_{c_0, h, \theta_{\text{cash}}} \left(E\left[U(c_0, c_1, h)\right] + \phi\right) \quad \text{s.t.} \quad c_0 + \theta_{\text{cash}} p_0 h \leq w, \quad \theta_{\text{cash}} \in [\underline{\theta}, 1], \\
&\qquad c_0 + \frac{c_1}{1+\rho} = w - \theta_{\text{cash}} p_0 h + \frac{y + \left[p_1 - (1 + r(1 - \theta_{\text{cash}}))\,(1 - \theta_{\text{cash}})p_0\right] h}{1+\rho}, \\
V_O^{\text{sal}} &= \max_{c_0, h, \theta_{\text{cash}}, \theta_{\text{sal}}} \left(E\left[U(c_0, c_1, h)\right] + \phi - \kappa\right) \\
&\quad \text{s.t.} \quad c_0 + \theta_{\text{cash}} p_0 h \leq w, \quad \theta_{\text{cash}}, \theta_{\text{sal}} \geq 0, \quad \theta_{\text{cash}} + \theta_{\text{sal}} \in [\underline{\theta}, 1], \\
&\quad c_0 + \frac{c_1}{1+\rho} = w - \theta_{\text{cash}} p_0 h + \frac{y + \left[p_1 - (1 + r(1 - \theta))\,(1 - \theta)p_0 - \theta_{\text{sal}}\left(p_0 + \gamma(p_1 - p_0)^+\right)\right] h}{1+\rho}
\end{aligned}$$

The renter optimizes housing and non-housing consumption, and the only period-0 need she funds with her wealth endowment is consumption. Her period-1 rent payment is set when she chooses housing tenure in period 0, so she bears no uncertainty.

The unassisted buyer faces two liquidity constraints. First, she must pay for a minimum share $\underline{\theta}$ of her home purchase in cash, making unassisted homeownership infeasible if her wealth w does not exceed the minimum down payment on a subsistence house $\underline{\theta} p_0 \underline{h}$. Second, because uncollateralized credit is not available, the agent must fund both consumption and a down payment out of her initial wealth. In Appendix C, we show that the user cost of owner-occupied housing in the unassisted homebuying problem is minimized at the down

payment share θ^*_{cash}, defined by

$$r(1-\theta^*_{\text{cash}}) + r'(1-\theta^*_{\text{cash}}) \cdot (1-\theta^*_{\text{cash}}) = \rho. \tag{1}$$

The left-hand side of equation 1 gives the marginal cost of an additional dollar of mortgage financing: interest at rate $r(1-\theta^*_{\text{cash}})$ on that dollar, plus the increase $r'(1-\theta^*_{\text{cash}})(1-\theta^*_{\text{cash}})$ in interest on the existing balance, as the higher loan-to-value ratio reprices the mortgage. The right-hand side gives the marginal benefit of a dollar of mortgage which, for an agent with slack in her period-0 budget constraint, is the return on a dollar of savings ρ. For an agent with a binding period-0 budget constraint, the marginal benefit of an additional dollar of mortgage financing is another dollar of *consumption*, whose benefit exceeds that of a dollar saved. In Appendix C, we show that an agent makes a cost-minimizing cash down payment if and only if her period-0 budget constraint carries no shadow value, which is the case whenever it is slack. This suggests that an agent's unassisted down payment share is a powerful tool for understanding whether homeownership entails a period-0 consumption strain. Applying this argument empirically requires that real-world borrowers who make proportionally small cash down payments are, as in the model, consumption-strained. Appendix Figure A.3 shows a strong relationship between down-payment share and non-housing wealth: the median first-time buyer at the 25th non-housing wealth percentile makes below a 3.5% down payment, which climbs gradually through the non-housing wealth distribution and reaches 20% at the 80th percentile. We return to this figure in Section 7.

The SAL relaxes both constraints discussed above. Although an SAL-assisted homebuyer must still fund any cash down payment out of initial wealth, a cash down payment is no longer necessary, as the shared appreciation loan can substitute for cash. Agents with wealth $w \leq \underline{\theta} p_0 \underline{h}$, who cannot buy unassisted, can therefore buy with Dream for All. Agents with $w > \underline{\theta} p_0 \underline{h}$, who could have bought unassisted, choose whether to make a cash down payment despite SAL availability depending on how comfortably they can fund their period-0 consumption needs out of their wealth endowment, the cost advantage of the SAL relative to cash, and their valuation of the SAL's risk-smoothing benefit. The SAL can relax a binding period-0 budget constraint for agents who would use up their entire period-0 wealth endowment between their period-0 consumption and a down payment when buying unassisted.

5.2 Characterizing the applicant population

Agents choose the tenure and financing alternative offering the highest indirect utility. Below, we characterize the population by their rankings of the three alternatives. Proposition 2 in

Appendix C formalizes the intuition developed below. We rely on assumptions (A1) and (A2), and further assume the SAL has appreciation-sharing parameter $\gamma < (p_H - p_L) \div (p_H - p_0)$ (satisfied for $\gamma = 1$ by the assumption that $p_L < p_0$).

We first consider the sub-population who can use the SAL at no hassle cost. Figure 4 Panel A, drawn at the parameter values estimated in Section 7, shows that these agents can be divided into three groups.[32] Two groups are *compliers*, who prefer to buy with the SAL. Some compliers, who we refer to as *marginal buyers*, would rather rent than buy unassisted. The share of applicants who are marginal buyers equals the causal effect of access to an SAL on homeownership: it is the share induced to become homeowners by SAL availability. Marginal buyers may rent rather than buy unassisted for two distinct reasons. First, marginal buyers with $w < \underline{\theta} p_0 \underline{h}$ may in fact prefer buying unassisted but are forced to rent by the minimum down payment. In Figure 4 Panel A, these agents are to the left of the dotted line. Second, marginal buyers who can buy unassisted, with $w > \underline{\theta} p_0 \underline{h}$, do not prefer homeownership enough to do so. These agents are in the blue region to the right of the dotted line representing the minimum down payment on a subsistence home, below the solid line delineating indifference between renting and buying unassisted, and above the dashed line delineating indifference between renting and buying with the SAL.

We note a stark prediction of our model: we show in Appendix C that, under the assumptions of Proposition 2, all marginal buyers strictly sacrifice housing consumption when switching from renting to buying, even with assistance from an SAL with appreciation-sharing parameter $\gamma \leq \gamma_{\text{fair}}$. This is because (A1) guarantees expected user cost of ownership is at least as high as rent.[33] Stone-Geary preferences imply that agents' housing consumption beyond subsistence is proportional to resources divided by this effective price, so the marginal buyer consumes less housing than a renter with the same resources.

Other compliers are *inframarginal takers*, whose fallback option is buying unassisted. Inframarginal takers have $w > \underline{\theta} p_0 \underline{h}$ and prefer homeownership enough to pay its higher cost. In Figure 4 Panel A, inframarginal takers are above the solid line delineating indifference between renting and buying unassisted and to the right of the dotted line delineating feasibility of unassisted homeownership. Although unassisted homeownership is by definition feasible, it may still strain consumption, particularly for inframarginal takers whose wealth narrowly exceeds the minimum down payment on a subsistence home.

[32] The partition holds for any parameter values satisfying the stated assumptions, but the threshold shapes depend on parameterization. The log utility assumption gives the solid line's vertical asymptote at the minimum down payment. The dashed line's flat segment arises if both a renter and an SAL-assisted buyer, who makes no cash down payment, consume their entire wealth endowments. The thresholds' slope beyond that depends on the weight on housing in utility β.

[33] Because the user cost of SAL-assisted ownership is highest in state L, its marginal-utility-weighted value strictly exceeds its expectation.

Even without hassle costs, some eligible agents may prefer renting to buying with the SAL because their preference to own does not compensate for the higher cost of ownership. These agents are *renter never-takers*. In Figure 4 Panel A, they lie below the dashed line delineating indifference between renting and buying with the SAL.

Figure 4 Panel B shows the two groups of agents for whom hassle costs are too high to use the SAL. In addition to the renter never-takers discussed above, who despite zero hassle cost rent by preference, there are also renter never-takers who would be marginal buyers but for their hassle costs. Finally, Panel B introduces a fourth group of agents, *buyer never-takers*, for whom preference for homeownership is high enough to compensate for the high cost of homeownership without the SAL and the hassle cost is too high to take it up.

5.3 Why did Dream for All increase homeownership?

Below we discuss the model's implications for judging which features of Dream for All explain the empirical results of Section 4: its elimination of the minimum cash down payment requirement; its relaxation of the homeownership-consumption tradeoff induced by the period-0 budget constraint; or merely the opportunity to consume owner-occupied housing at an effectively subsidized price. Figure 5 depicts each scenario described below.

Recall that the effect of Dream for All on homeownership equals the size of the marginal buyer population. If the elimination of a minimum cash down payment entirely explains the causal effect of Dream for All on homeownership documented in Section 4, then all marginal buyers must have wealth below the dotted line in Figure 5 Panel A representing the minimum down payment on a subsistence house, so that they could not buy unassisted. Every marginal buyer is poorer than every inframarginal taker who, by definition, must have enough wealth to buy unassisted. Although wealth is not directly observed, we expect it to be correlated with income, consumer credit utilization, and delinquency. If Dream for All exclusively increased homeownership through elimination of the minimum cash down payment constraint, we should see little overlap in marginal buyers' and inframarginal takers' distributions of characteristics correlated with wealth.

Even if exemption from a minimum cash down payment was not primarily responsible for the homeownership effect, Dream for All may have increased homeownership by relaxing another binding constraint: the requirement to fund both a cash down payment and consumption at the same time. In this scenario, many marginal buyers have wealth above the minimum down payment on a subsistence house, and, therefore, their wealth distribution overlaps with that of inframarginal takers, as in Figure 5 Panel B. Although they both prefer Dream for All, marginal buyers and inframarginal takers at the same wealth level have

different fallback options because the latter have higher preference for homeownership over renting. Holding wealth constant, inframarginal takers make the same consumption and financing choices as would marginal buyers because preference for homeownership and utility from housing and non-housing consumption are additively separable in indirect utility. This delivers a powerful test: if Dream for All primarily increased homeownership by relaxing the homeownership-consumption tradeoff, inframarginal takers with the same observable resources as marginal buyers strain to afford homeownership, as evidenced by down payment shares below the cost-minimizing share. If inframarginal takers with the same observable resources as marginal buyers sacrifice consumption when buying unassisted, we can blame these consumption sacrifices for deterring marginal buyers from unassisted homeownership.

A third possibility is that Dream for All did not alleviate any binding constraint, but that applicants took it up only because it offered subsidized housing consumption under homeownership. Like the previous scenario, this scenario could feature substantial overlap between marginal buyers' and inframarginal takers' wealth distributions. However, wealth levels of both groups are higher in this scenario, as in Figure 5 Panel C, and inframarginal takers make cost-minimizing cash down payments. We could still see an elimination of the cash down payment if Dream for All is cheaper than cash, as well as a reduction in the mortgage share of the purchase price. However, this scenario requires that Dream for All utilization increases housing consumption for inframarginal takers. In Appendix C, we show that, under the assumptions of Proposition 2, for any positive housing consumption weight in utility $\beta > 0$, an inframarginal taker whose period-0 budget constraint does not bind with or without the SAL consumes strictly more housing with it, at any appreciation-sharing parameter at which she takes up the SAL.

The following section leverages the random allocation of Dream for All loans to distinguish between these three scenarios by comparing the characteristics of marginal buyers and inframarginal takers, and estimating the effects of Dream for All utilization separately for these two complier sub-groups.

6 Mechanisms: empirical evidence

Section 4 conceives of a Dream for All offer as a treatment and measures its effect on homeownership. To accommodate the theoretical framework developed in Section 5, this section adapts a multinomial instrumental variables model developed by Kline and Walters (2016) which reframes a Dream for All offer as an instrument that induces substitution between three treatments: renting, buying unassisted, and buying with Dream for All.

6.1 Characterizing the applicant population non-parametrically

Econometric framework Let $\text{LOTTERY}_i \in \{0, 1\}$ denote whether applicant i receives a Dream for All offer, and let $D_i(\ell) \in \{\text{Buy with DFA}, \text{Buy unassisted}, \text{Rent}\}$ denote household i's potential tenure and financing choice for offer status $\text{LOTTERY}_i = \ell$. Observed treatment status is $D_i = D_i(\text{LOTTERY}_i)$. Let $Y_i(d)$ be applicant i's potential outcome under treatment $d \in \{\text{Buy with DFA}, \text{Buy unassisted}, \text{Rent}\}$, with observed outcomes Y_i equal to $Y_i(D_i)$. This notation embeds an exclusion restriction: Dream for All offers LOTTERY_i affect outcomes only through the tenure and financing choice D_i.[34]

We do not expect a Dream for All offer to cause applicants to switch between renting and buying unassisted, nor do we expect a Dream for All offer to reduce the likelihood that an applicant buys with Dream for All. We therefore impose the monotonicity restriction

$$D_i(1) \neq D_i(0) \Rightarrow D_i(1) = \text{Buy with DFA}$$

which states that any applicant who changes their tenure and financing choice because they receive an offer does so to buy with Dream for All. This rules out an applicant who applies on a whim without intending to buy, but begins to search for homes upon receiving an offer, learns that she prefers to buy unassisted, and declines the Dream for All offer. We regard such belated discoveries as unlikely because applying for Dream for All required completion of an eight-hour homebuyer education course and mortgage pre-approval.

The monotonicity restriction partitions applicants into the groups discussed above: marginal buyers, inframarginal takers, renter never-takers, and buyer never-takers. Marginal buyers are applicants for whom $D_i(0) = \text{Rent}$ and $D_i(1) = \text{Buy with DFA}$; they rent when they do not receive a Dream for All offer, and buy with Dream for All when they do. Analogously, inframarginal takers are characterized by $D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}$. Renter never-takers and buyer never-takers have $D_i(0) = D_i(1) = \text{Rent}$ and $D_i(0) = D_i(1) = \text{Buy unassisted}$, respectively. The framework also allows for the possibility of always-takers, who buy using Dream for All regardless of offer status, but there were no such applicants.[35]

[34] This model generalizes the canonical Imbens and Angrist (1994) binary instrument, binary treatment instrumental variables model with heterogeneous treatment effects. That model would specify a two-dimensional vector of possible treatments $\{\text{Buy with DFA}, \text{Don't buy with DFA}\}$. The trinomial model presented in this section splits the non-focal "treatment" into renting and buying unassisted, and so splits the binary model's compliers into two *principal strata* (Frangakis and Rubin, 2002) defined by what an applicant would do without an offer.

[35] Because we observe outcomes at multiple time horizons, an applicant's complier status can change over time. For example, an applicant who would buy immediately if offered a loan but would rent initially and then buy without assistance if not offered would be classified as a marginal buyer at short horizons and an inframarginal taker later on. The analysis below uses outcomes measured six quarters after the program lottery, by which point applicants' tenure and financing choices are fairly stable.

Figure 6 shows how to identify the applicant population shares of each of the four types. As noted in subsection 5.2, the marginal buyer share is the causal effect of Dream for All on homebuying, and is identified as the difference between the shares of the treatment and control groups who buy a home. It is also identified by the difference between these shares' complements, the renter shares in the treatment and control group. We employ this complementary identification strategy when measuring the size and characteristics of marginal buyers using two-stage least squares regressions, explained below. Buyer and renter never-taker shares are identified by the shares of applicants who receive a Dream for All offer, decline it, and continue to buy unassisted or rent, respectively.[36] These groups make up 8.0% and 29.1% of the applicant population, respectively. The difference between the share of the control group and the treatment group who buys unassisted identifies the inframarginal taker share, which is 20.1%. Intuitively, we use the treatment group to judge which proportion of the control group buyers would have bought unassisted had they received an offer. The plot also shows that the total complier share – the share of applicants induced to buy with Dream for All by a Dream for All offer – is 62.9%. This is the sum of the marginal buyer and inframarginal taker population shares, as noted in subsection 5.2.

The cross-group differencing described above can be implemented by the regressions

$$\begin{aligned} \text{RENT}_i &= \alpha_1 + \beta_1 \text{LOTTERY}_i + \mathbf{R}_i\Gamma_1 + \varepsilon_{i1} \\ \text{BUY UNASSISTED}_i &= \alpha_2 + \beta_2 \text{LOTTERY}_i + \mathbf{R}_i\Gamma_2 + \varepsilon_{i2} \end{aligned}$$

where $\mathbf{R}_i$ is the vector of randomization strata. The coefficient β_1 is the difference between the treatment and control groups' renter shares, controlling for randomization stratum, so $-\beta_1$ is the marginal buyer population share, and $-\beta_2$ analogously gives the inframarginal taker population share. Kline and Walters (2016) also show how to identify average characteristics of the two complier sub-groups. For example, we can measure inframarginal takers' average income by estimating the two-stage least squares system

$$\begin{aligned} \text{Reduced form:} \quad & \text{Income}_i \times \text{BUY UNASSISTED}_i = \zeta_2 + \theta_2 \text{LOTTERY}_i + \mathbf{R}_i\Omega_2 + \omega_{i2} \\ \text{First stage:} \quad & \text{BUY UNASSISTED}_i = \alpha_2 + \beta_2 \text{LOTTERY}_i + \mathbf{R}_i\Gamma_2 + \varepsilon_{i2} \end{aligned} \tag{2}$$

Intuitively, the reduced form coefficient $-\theta_2$ identifies the total income of inframarginal takers as the difference between the total income of treatment group unassisted buyers (buyer

[36] Here and throughout the paper, we say that an applicant who receives but does not use an offer *declines* the offer. Some applicants who received and did not use the offer might be better described as having failed to use it if they, for example, were prevented from using their offer by the time limit or other challenges searching for a home. Given data limitations, we have little ability to distinguish between declining and failure to use the loan.

never-takers) from control group buyers (inframarginal takers plus buyer never-takers). Dividing the reduced form coefficient $-\theta_2$, which gives inframarginal takers' total income, by their population size, identified by the first stage coefficient $-\beta_2$, gives their average income. This procedure works analogously for marginal buyers, and also for any covariate, including indicators for e.g., earnings above a certain threshold. The procedure also works for outcomes: replacing Income_i in the reduced-form regression above with a Mortgage size_i would give the average mortgage size of inframarginal takers when buying unassisted.

Results Panel A of Figure 7 shows that the income distributions of marginal buyers and inframarginal takers overlap substantially. This pattern starkly rejects the hypothesis that all marginal buyers are poorer than all inframarginal takers, which, as discussed in subsection 5.3, would have to be true if elimination of the minimum down payment constraint explained the entire effect of Dream for All on homeownership. Panel B shows substantial overlap in the distribution of Dream for All-contingent mortgage pre-approvals, which further supports the idea that marginal buyers and inframarginal takers had similar resources because mortgage pre-approvals were issued not only on the basis of income but also assets. The overlap in mortgage pre-approvals also addresses an omission in our model, namely, heterogeneous minimum housing needs $\underline{h}$. A marginal buyer could have the same income and wealth as an inframarginal taker and still be constrained if they had higher housing needs. To the extent that Dream for All-contingent pre-approvals proxy housing needs, Panel B of Figure 7 suggests this is not the case, bolstering the case for our non-overlapping incomes test.

Appendix Table A.3 shows that marginal buyers and inframarginal takers resemble one another not only on these, but many other dimensions. Although average incomes and mortgage pre-approvals differ significantly, racial and ethnic distributions, subordinate loan pre-approvals, credit scores, credit limits, auto loans, credit cards, and student loans are not statistically distinguishable across the two groups. Marginal buyers are significantly more likely to be female (+8 percentage points) and have significantly higher average revolving credit utilization (+$3,100), but they are also older on average (+1.6 years) and have significantly fewer delinquencies since 2020 (-5.6 percentage points). Taken together, marginal buyers appear neither obviously poorer nor less qualified for a conventional mortgage than inframarginal takers, casting further doubt on the hypothesis that elimination of the minimum down payment constraint or relaxed lending standards were responsible for Dream for All's impact on homeownership.

We now consider the second hypothesis, that Dream for All increased homeownership by relaxing the homeownership-consumption tradeoff embodied by the model's period-0 budget constraint, in which unassisted buyers must simultaneously make a down payment and

fund their consumption. Recall from subsection 5.3 that we can assess this hypothesis by checking whether inframarginal takers whose wealth is similar to that of marginal buyers strain consumption under homeownership, as evidenced by cash down payments below the cost-minimizing share. Figure 8 implements this exercise, showing the distribution of inframarginal takers' down payments when buying unassisted. Inframarginal takers exhibit clear consumption strain when buying unassisted: over 40% make down payments of 3.5%, the minimum down payment on a standard government-insured mortgage offered by the Federal Housing Administration, or lower, which is allowed under certain special government-insured mortgages offered by the Veterans Administration and the US Department of Agriculture. Nearly three quarters make down payments below 20%, which is generally regarded as the cost-minimizing down payment share.[37] Intuitively, inframarginal takers making sub-optimal down payments tolerate higher borrowing costs, including interest and mortgage insurance premiums, to preserve consumption at origination. Taking seriously both the model and evidence from the previous exercise that marginal buyers and inframarginal takers have similar resources, we can infer that, had marginal buyers bought unassisted, their consumption would also have been strained. This suggests that Dream for All indeed increased homebuying by offering an opportunity to buy with less strain on consumption.

Another possibility is that applicants used Dream for All not to relax an ownership-consumption tradeoff, but instead to increase housing consumption, perhaps by increasing mortgage borrowing.[38] Figure 9 assesses this possibility by comparing inframarginal takers' average mortgage size when they buy unassisted to their Dream for All-contingent pre-approvals. On average, inframarginal takers took out mortgages of $556K when buying unassisted, compared to average pre-approvals of $519K. Appendix Figure A.5 shows that these contingent pre-approvals do a good job of predicting mortgage utilization when buying with Dream for All, although there is some mean reversion. This suggests the difference between Dream for All-contingent pre-approval and unassisted mortgage size can be read as a measure of the causal effect of Dream for All utilization on mortgage size. Read this way, the plot suggests that Dream for All utilization reduces mortgage borrowing by $37K, which is the difference between the two bars, shown in gray.

[37] Consistent with this, Appendix Figure A.3 shows that the median first-time buyer at the 95th percentile of the non-housing wealth distribution makes a 20% down payment.

[38] The opportunity to increase access to mortgage financing and "buy more house" was explicitly advertised by one CalHFA partner lender, John King, in a YouTube tutorial video. Appendix Figure A.4 Panel A shows a screenshot of Mr. King walking prospective Dream for All buyers through scenarios to "increase their buying power" with Dream for All. To Mr. King's credit, he subsequently walks the audience through scenarios in which Dream for All can lower monthly payments on the same house.

6.2 Effects of shared appreciation loan utilization

Econometric framework The exercise above comparing inframarginal takers' mortgages when buying unassisted to Dream for All-contingent pre-approvals can be thought of as characterizing the difference below, which Kline and Walters (2016) term a "subLATE"

$$E\left[Y_i(\text{Buy with DFA}) - Y_i(\text{Buy unassisted}) \mid D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}\right],$$

where Y_i is mortgage size, so $Y_i(\text{Buy with DFA})$ and $Y_i(\text{Buy unassisted})$ are potential mortgage sizes when buying with DFA and unassisted, respectively. The conditional expectation gives the average difference among inframarginal takers, whose potential treatments are $D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}$. Average unassisted mortgage size, $E\left[Y_i(\text{Buy unassisted}) \mid D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}\right]$, is identified by the two-stage least squares procedure discussed above, and the strong predictive relationship between Dream for All-contingent pre-approvals and mortgage utilization among Dream for All users suggests this characteristic may be a good proxy for inframarginal takers' average mortgage utilization when buying with Dream for All,
$E\left[Y_i(\text{Buy with DFA}) \mid D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}\right]$.

This approach is limited for two reasons. First, pre-approvals are noisy proxies for mortgage size and, in particular, inframarginal takers have slightly higher average pre-approvals than marginal buyers (Appendix Table A.3), so we might expect inframarginal takers' mortgages with Dream for All to fall short of their pre-approvals, in which case the \$37K figure understates the effect of Dream for All utilization on mortgage size. Second, this method is not available for other outcomes of interest. Measuring effects on outcomes not pre-registered in applications requires an alternative approach to proxying the average potential outcome under the Buy with DFA treatment.

Kline and Walters (2016) note that mean potential outcomes of the different complier sub-groups under the focal treatment (here, buying with Dream for All) are not separately identified. To see this, note that we can write the complier potential outcome mean as a weighted average of subgroup-specific potential outcomes

$$\begin{aligned}
&\underbrace{E\left[Y_i(\text{Buy with DFA}) \mid D_i(0) \neq \text{Buy with DFA}, D_i(1) = \text{Buy with DFA}\right]}_{\text{Overall complier mean}} \\
&\quad = S_m \underbrace{E\left[Y_i(\text{Buy with DFA}) \mid D_i(0) = \text{Rent}, D_i(1) = \text{Buy with DFA}\right]}_{\text{Marginal buyer mean}} \\
&\qquad + (1 - S_m) \underbrace{E\left[Y_i(\text{Buy with DFA}) \mid D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}\right]}_{\text{Inframarginal taker mean}}
\end{aligned}$$

where S_m is the marginal buyer share of compliers. The left-hand side of the above expression is identified simply as the average mortgage size among Dream for All users, and the shares S_m are identified by the complier population size techniques described in the previous subsection. However, the contributions of the two sub-group averages are unknown.

The simplest way around this obstacle is the "representation in observables" (RIO) assumption of Angrist, Santos and Tecchio (2026) (henceforth AST): among compliers, the conditional mean of the potential outcome does not depend on complier type. In our setting, buying with Dream for All is the only treatment reached by both complier types, so RIO reduces to the restriction that marginal buyers and inframarginal takers have the same mean potential outcome under Dream for All-assisted buying. RIO implies that a marginal buyer and an inframarginal taker will, on average, take out the same size mortgage when buying with Dream for All, so the overall average mortgage size among compliers can stand in for the complier sub-group specific averages. Then the inframarginal taker subLATE is

$$\underbrace{E\left[Y_i(\text{Buy with DFA}) - Y_i(\text{Buy unassisted}) \mid D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}\right]}_{\text{Inframarginal taker subLATE}}$$
$$= \underbrace{E\left[Y_i(\text{Buy with DFA}) \mid D_i(0) \neq \text{Buy with DFA}, D_i(1) = \text{Buy with DFA}\right]}_{\text{Complier mean outcome under DFA}}$$
$$- \underbrace{E\left[Y_i(\text{Buy unassisted}) \mid D_i(0) = \text{Buy unassisted}, D_i(1) = \text{Buy with DFA}\right]}_{\text{Inframarginal taker mean outcome under unassisted buying}}$$

The assumption could be relaxed by conditioning on observables: rather than assuming unconditional equality, we could assume equality among marginal buyers and inframarginal takers with the same characteristics. Because Appendix Table A.3 shows relatively few differences in observed characteristics across these two groups, this adjustment would yield very similar subLATE estimates. Despite the groups' similarity on observables, the RIO assumption would be violated if the two complier subgroups differed on unobservable attributes which determine applicants' counterfactual choices when not offered Dream for All *and* affected their outcomes when they use Dream for All. In particular, the assumption is compatible with the stylized model of Section 5 to the extent that marginal buyers' unassisted tenure choice differs from that of inframarginal takers due to taste for homeownership, which by additive separability does not affect agents' financing choices or non-housing and housing consumption. It is incompatible with the model if the two groups' wealth differs on average, as wealth affects both counterfactual tenure and behavior.

This possibility motivates a second identification strategy which allows for unobserved taste for each treatment to affect potential outcomes, by adapting the trinomial selection model of Kline and Walters (2016). The selection model, akin to a two-dimensional Heckit,

assumes unobserved taste for buying with and without Dream for All is distributed according to a bivariate normal distribution, and allows the resulting taste shock estimates to affect potential outcomes under renting, buying unassisted, and buying with Dream for All.

We estimate subLATEs using both approaches. Our implementation of AST follows their replication of Kline and Walters's (2016) re-analysis of the Head Start Impact Study. We construct a recentered instrument

$$\widetilde{\text{LOTTERY}}_i = \frac{\text{LOTTERY}_i - \hat{p}(X_i)}{\hat{p}(X_i) \cdot (1 - \hat{p}(X_i))}$$

where $\hat{p}(X_i)$ are probit predictions of lottery offer receipt from pre-determined covariates.[39] We estimate subLATEs by differencing across estimated average potential outcomes under buying with Dream for All, renting, and buying unassisted. Given the recentered instrument $\widetilde{\text{LOTTERY}}_i$, we estimate the three two-stage least squares systems

$$\begin{aligned} Y_i \times \text{RENT}_i &= \lambda_1 + \mu_1 \widetilde{\text{LOTTERY}}_i + \mathbf{R}_i \Xi_1 + e_{i1} \\ \text{RENT}_i &= \delta_1 + \pi_1 \widetilde{\text{LOTTERY}}_i + \mathbf{R}_i \Delta_1 + u_{i1} \end{aligned} \tag{3}$$

$$\begin{aligned} Y_i \times \text{BUY UNASSISTED}_i &= \lambda_2 + \mu_2 \widetilde{\text{LOTTERY}}_i + \mathbf{R}_i \Xi_2 + e_{i2} \\ \text{BUY UNASSISTED}_i &= \delta_2 + \pi_2 \widetilde{\text{LOTTERY}}_i + \mathbf{R}_i \Delta_2 + u_{i2} \end{aligned} \tag{4}$$

$$\begin{aligned} Y_i \times \text{BUY WITH DFA}_i &= \lambda_3 + \mu_3 \widetilde{\text{LOTTERY}}_i + \mathbf{R}_i \Xi_3 + e_{i3} \\ \text{BUY WITH DFA}_i &= \delta_3 + \pi_3 \widetilde{\text{LOTTERY}}_i + \mathbf{R}_i \Delta_3 + u_{i3} \end{aligned} \tag{5}$$

Mirroring the average complier characteristics identification strategy described around the system of equations 2, the IV coefficients $\mu_1 \div \pi_1$, $\mu_2 \div \pi_2$, and $\mu_3 \div \pi_3$ identify average potential outcomes under renting for marginal buyers, buying unassisted for inframarginal takers, and buying with DFA for both groups, respectively.[40] We calculate subLATEs as pairwise differences between IV coefficients: for example, the effect of Dream for All utilization on inframarginal takers is estimated as $(\mu_3 \div \pi_3) - (\mu_2 \div \pi_2)$. We use the nonparametric block bootstrap to calculate standard errors. Appendix D describes our implementation of the selection model, which closely follows Kline and Walters (2016), and presents results. The two methods give qualitatively similar estimates for our main outcomes, although the

[39] Intuitively, this instrument adjusts each applicant's offer status for their predicted probability of receiving an offer, reweighting to account for variation across covariate cells. Because lottery offers were randomly assigned within region, the recentering amounts to adjusting offer status for region-specific offer rates.

[40] Systems 3 and 4 do the same differencing across treatment and control groups discussed in subsection 6.1. System 5 simply gives the average characteristics of compliers; since the experiment has one-sided non-compliance, this simply averages over Dream for All buyers in the treatment group.

selection model's financing effects are larger and less precisely estimated.

Results Table 1 shows estimates of effects of Dream for All utilization on inframarginal takers' home purchase terms. Panel A shows effects on financing. Dream for All utilization reduces mortgage size by $74K on average, and reduces monthly mortgage payments by $342. In relative terms, Dream for All reduced mortgage size and monthly payments by 13% and 8%, respectively. The mortgage size reduction corresponds to a first mortgage loan-to-value ratio reduction of 9.3 percentage points. However, buying with Dream for All increases combined loan-to-value (CLTV) ratios – total borrowing relative to the home's purchase price, including the shared appreciation loan – by 9.9 percentage points. Appendix Figure A.6 aids interpretation of these effects. When buying unassisted, the average inframarginal taker finances 89.7% of her home purchase with a mortgage and makes a 10.3% cash down payment. Using Dream for All, her mortgage borrowing falls to about 80% of the home's purchase price, her cash contribution falls to less than one percent, and the Dream for All loan makes up the difference, funding 19.2% of the purchase on average. These average numbers mask heterogeneity in counterfactual down payments – as shown in Figure 8, the modal inframarginal taker makes a minimum cash down payment when buying unassisted, while over a quarter make down payments of 20% or higher. Consistent with the mortgage size and down payment reductions documented in Panel A, Panel B shows no evidence of increases in housing consumption across multiple dimensions, including price paid, building size, and number of rooms. For price paid, the estimates are precise enough to rule out a home price increase greater than 0.3%.

A homebuyer buys not just a property but also a bundle of neighborhood amenities, so restricting attention to property characteristics could miss important housing consumption effects. Figure 10 shows effects of Dream for All utilization on several dimensions of neighborhood quality for both inframarginal takers and marginal buyers. We measure outcomes as standard deviations relative to pre-lottery mean in the control group, and re-orient each measure so that a lower score corresponds to a lower-amenity neighborhood. Red dots reflect differences between the neighborhood inframarginal takers would buy in with Dream for All assistance and the neighborhoods where they would buy unassisted. Blue dots show differences between the neighborhoods where marginal buyers would purchase with Dream for All assistance and the neighborhoods where they would rent. Across all the dimensions of neighborhood quality for which we measure effects, we cannot reject the hypothesis that Dream for All caused zero effect for inframarginal takers relative to their unassisted buying counterfactuals, consistent with effects on housing consumption shown in Table 1.

In contrast, Dream for All utilization decreased neighborhood quality for marginal buy-

ers on several dimensions, including the Opportunity Atlas' measure of upward mobility (Chetty, Friedman, Hendren, Jones and Porter, 2026), the Childhood Opportunity Index (Noelke, McArdle, Baek, Huntington, Huber, Hardy and Acevedo-Garcia, 2020), the Area Deprivation Index (Kind and Buckingham, 2018), the fraction of adults aged 25+ with at least a college degree, mean third grade math test scores, the US Housing and Urban Development Department's School Index, median neighborhood rents, job density, total and high-paying jobs, and population density. Although some of these effects are statistically indistinguishable from effects on inframarginal takers, they collectively suggest a tradeoff between homeownership and housing consumption: even with assistance from Dream for All, marginal buyers sacrifice neighborhood quality in order to achieve homeownership. These effects are consistent with the predictions of the theoretical framework presented in Section 5, in which applicants face higher costs of ownership than renting and therefore make housing consumption sacrifices in order to gain the utility boost from owning one's home.

Complementary evidence on housing consumption Results presented in Table 1 and Figure 10 suggest that Dream for All utilization did not increase housing consumption. Combined with Appendix Table A.1, which suggests that Dream for All likely represented a substantial expected cost reduction for the modal inframarginal taker who would make a 3.5% down payment, these results imply a very small housing consumption elasticity with respect to financing costs.

Appendix Figure A.7 provides an independent test of the effect of financing costs on housing consumption using a program which caused a discontinuous jump in Dream for All's cost at an earnings threshold. Recall from subsection 2.2 that applicants earning above 80% of Area Median Income in the county where they purchased owed the financing share of appreciation, whereas borrowers earning below the threshold owed just three quarters of the financing share.[41] We test for declines in Dream for All take-up and housing consumption around the 80% threshold using binned scatterplots constructed following Calonico, Cattaneo and Titiunik (2015), overlaid with linear fits. Manipulation tests following Cattaneo, Jansson and Ma (2018), shown in Appendix Figure A.7 Panel A, fail to reject continuity at the 80% cutoff for both the full applicant sample and the sub-sample of Dream for All takers. The binned scatter plots in Panels B–D do not show any evidence of a take-up or housing consumption response to increased costs.

[41] For example, a homebuyer who financed a 20% down payment for a $500K home using a Dream for All loan and later sold for $750K would pay $12.5K less in appreciation if she earned 79% of AMI in the county where she purchased than if she earned 81%.

Projecting Dream for All's effect on default risk Our results suggest that Dream for All benefitted applicants by enabling them to become homeowners with lower down payments and monthly mortgage payments. A natural question is whether these benefits are likely to come at the expense of mortgage lenders and/or taxpayers, which could be the case if Dream for All-assisted buyers default on their obligations.

Theoretically, the effect of Dream for All utilization on inframarginal takers' likelihood of mortgage default is ambiguous. Table 1 shows that Dream for All utilization did not affect inframarginal takers' home purchase size, reduced first mortgage loan-to-value ratios and monthly mortgage payments, and increased CLTVs. CLTV increases are thought to increase the mortgage default risk, as a decline in the value of a home relative to its associated mortgage reduces the return to paying off the mortgage for a homebuyer with little home equity. On the other hand, high monthly mortgage payments can increase the risk of liquidity-driven default, whereby a borrower stops paying her mortgage because she has insufficient income or savings to cover her payment.

Appendix Table A.4 uses estimates of the effects of monthly mortgage payments and CLTV from the literature to forecast the effect of Dream for All utilization on default risk. Panel A reproduces the estimates from Table 1. Panel B shows relevant estimates from the literature. Using a proportional hazard model, Fuster and Willen (2017) estimate that a 1% reduction in monthly payments reduces default risk by 1.6%, and a one percentage point increase in CLTV raises default risk by 1.85%. Ganong and Noel (2020) use a linear model to estimate default effects: they find that a 1% reduction in monthly payments reduces default risk by 1.2%, and a one percentage point increase in CLTV lowers default risk by 0.55%. Panel C shows the implied change in default risk, combining our estimates with the default effects. We assume, conservatively, that the 8.3% reduction in the monthly payment inclusive of taxes and insurance applies to principal and interest, the payment concept in both papers.[42] We also treat the statistically insignificant negative Ganong and Noel (2020) CLTV effect as a zero. Under the Fuster and Willen (2017) estimates, Dream for All utilization leaves the default hazard approximately unchanged (hazard ratio 1.05); under the Ganong and Noel (2020) estimates, it reduces the hazard by 10% (0.90). We note that these calculations are speculative: they extrapolate default effect estimates from settings in which homebuyers were deep underwater to effects on mortgage terms at origination, and all the estimates in the table are reported with uncertainty.[43] Nonetheless, they cast doubt

[42]Dream for All utilization should not affect taxes and insurance, so the reduction in principal and interest is larger than 8.3%.

[43]In particular, pairing the smallest payment reduction ($125) with the largest CLTV increase (10.6 percentage points) from the 95% confidence intervals in Table 1 gives net hazard ratios of 1.16 and 0.96 when using Fuster and Willen (2017) and Ganong and Noel (2020) estimates, respectively.

on the idea that Dream for All will starkly increase default risk.

We have less to say about the effect of Dream for All utilization on marginal buyers' mortgage default risk. On the one hand, Dream for All utilization must increase mortgage default risk for marginal buyers, as they would not have any mortgage debt without Dream for All. On the other hand, Figure 7 and Appendix Table A.3 suggest that marginal buyers are, on observable metrics, not much less qualified for mortgages than inframarginal takers.

Discussion This section estimates the characteristics of marginal buyers, applicants induced to buy homes because they received a Dream for All offer, and compares them to inframarginal takers, who could have bought unassisted but preferred Dream for All. We also estimate effects of Dream for All utilization on financing and housing consumption. Our findings are consistent with the idea that marginal buyers could have made above-minimum cash down payments and qualified for mortgages without Dream for All assistance. When buying unassisted, the modal inframarginal taker made the minimum allowed down payment, and the overwhelming majority made down payments less than 20%. This suggests that buying without assistance strains inframarginal takers' budgets. Because marginal buyers resemble inframarginal takers on observable characteristics associated with wealth, we infer that marginal buyers, despite being able to qualify for a mortgage, would have been strained under unassisted homeownership. We show that inframarginal takers use Dream for All to alleviate the strain of homeownership, decreasing mortgage borrowing and cash down payments, rather than increasing housing consumption. Finally, we document decreases in neighborhood quality for marginal buyers, consistent with a prediction stated in Section 5.

These results align with the second scenario presented in Section 5, in which Dream for All increases homeownership by relaxing the period-0 budget constraint, which requires buyers to fund both a cash down payment and consumption out of the same resources. Elimination of the down payment constraint may have played a secondary role in mediating the causal effect of Dream for All on homeownership, as marginal buyers are modestly negatively selected on some characteristics (e.g., income, mortgage pre-approval) correlated with wealth. The results are incompatible with the third scenario, in which Dream for All relaxes no binding constraint and merely offers subsidized owner-occupied housing.

The following section uses this evidence to answer quantitative questions: what would applicants have been willing to pay for Dream for All assistance, and how does their willingness to pay compare to the expected fiscal cost the government will incur?

7 Policy implications

This section parametrizes the model presented in Section 5 and uses the evidence accumulated thus far to inform calibration choices and estimate the joint distribution of wealth, preference for homeownership, and hassle costs. We use the estimated model to conduct a welfare evaluation of Dream for All, employing the *marginal value of public funds* (MVPF) metric weighing applicants' willingness to pay (WTP) for assistance against the program's expected fiscal cost. While the model's parsimony is useful for giving analytical predictions, it may miss important features of reality that affect applicants' WTP for assistance or its expected cost. We therefore implement a battery of robustness tests to probe the sensitivity of our conclusions to our admittedly strong assumptions. Finally, we compare the effects and welfare gains from Dream for All to counterfactual forms of assistance that could have been given to the applicants.

7.1 Parametrization and estimation

We calibrate preference parameters and features of the financial environment. Our calibration choices are shown in Panels A and B of Table 2. We set both the scale of income and the period-0 purchase price of housing at 1. The mortgage rate schedule, shown in Appendix Figure A.8, is chosen so that the cost-minimizing down payment share is 20% and the mortgage interest rate at maximum leverage exceeds the household return on savings. We enforce a realistic minimum down payment requirement: agents buying unassisted must pay at least 3.5% of the home purchase price in cash. The government bond coupon ρ (which also serves as the household return on savings) is set to 5%.[44] We calibrate the price process to annualized repeat-sale returns in California, 1988–2023, weighting each spell by its purchase price: $\pi = 0.76$ is the share of purchases held at least five years which are resold above their purchase prices, and $p_H = 1.048$ is one plus the median annualized return on purchases held at least five years resold above purchase price. We set $p_L = 0.888$ to one plus the median annualized return among spells of the 2005-08 home purchase cohorts in which sale prices fell below purchase prices. Because some calibration choices are rather arbitrary, we extensively probe the sensitivity of our conclusions to our baseline assumptions.

The zero intensive-margin effect of Dream for All utilization on inframarginal takers' housing consumption suggests a high minimum housing consumption need $\underline{h}$ and small

[44] This matches the interest cost on California's federally taxable general obligation bonds in April 2024 (4.99–5.10%); tax-exempt bonds sold at 3.53% in the same offering (California State Treasurer's Office, 2024). We use the taxable rate because the gap is a federal subsidy rather than a saving to society. A cost of capital at the high end of the state's range also leans against the SAL, partially compensating for the baseline model's government being risk-neutral over payoffs correlated with the macroeconomy.

marginal propensity to consume housing out of disposable resources β; we therefore assume the minimum house has a purchase price of $\underline{h} = 4.5$ times the household's exogenous income, and her marginal propensity to consume housing is $\beta = 0.01$. In Section 6, we document that Dream for All utilization reduces inframarginal takers' monthly mortgage payments by \$342, roughly the reduction that a 1 percentage point lower mortgage rate would produce on their counterfactual \$556,000 mortgage, and can reject a housing consumption response greater than 0.3 percent (Table 1).[45] Our calibration choices imply, for the median inframarginal taker, that the elasticity of housing consumption with respect to lifetime resources is about 0.05; a 1 percentage point increase in the mortgage rate reduces housing consumption by about half a percent; and a \$100,000 wealth windfall raises it by about 3 percent. These calibration choices are consistent with small documented intensive-margin housing consumption responses to mortgage financing costs (DeFusco and Paciorek, 2017; Bhutta and Ringo, 2021), but starkly at odds with income elasticities of housing demand estimated in the urban economics literature (Aguiar and Bils, 2015; Diamond and Gaubert, 2022; Finlay and Williams, 2025) and with the housing consumption response to an experimental cash transfer (Bartik, Rhodes, Broockman, Krause, Miller and Vivalt, 2026).[46] We speculate that differences in time horizons may help explain the divergence across literatures: estimates in the urban economics literature are intended to reflect long-run responses to permanent differences in resources, whereas estimates in finance reflect responses around a single (albeit durable) decision that households may find difficult to adjust instantaneously.[47] To the best of our knowledge, we are the first to note this tension. Although reconciliation is beyond the scope of this paper, we regard it as an interesting subject for future research.

We specify a bivariate normal distribution for log wealth and log preference for homeownership, indexed by distribution-specific means and variances $\mu_w, \mu_\phi, \sigma_w^2, \sigma_\phi^2$ and a correlation parameter ϱ. We assume the conditional probability of inability to use assistance given

[45] The upper bound of the 95 percent confidence interval for the effect on price paid in Table 1, \$2,201, is 0.34 percent of the counterfactual price of \$639,882. The monthly payment on a 30-year fixed-rate mortgage is $M \cdot \frac{r/12}{1-(1+r/12)^{-360}}$ for principal M and annual rate r; at $M = \$555{,}541$, lowering r from 6.5 to 5.5 percent reduces the payment by \$357, and from 7 to 6 percent by \$365.

[46] Bunching estimates around the conforming loan limit in DeFusco and Paciorek (2017) suggest that a 1 percentage point mortgage rate increase reduces mortgage size by 2-3 percent. Using an RD in time, Bhutta and Ringo (2021) find that a surprise 50 basis point cut in mortgage insurance premiums raised purchase prices by 1.3 percent. IV estimates using the Consumer Expenditure Survey and the PSID in Aguiar and Bils (2015) and Finlay and Williams (2025) give expenditure elasticities of housing services of 0.92 and 0.75, respectively; Diamond and Gaubert (2022) survey this literature. Bartik, Rhodes, Broockman, Krause, Miller and Vivalt (2026) randomize a three-year, \$1,000-per-month transfer and find that housing spending rises 0.7 percent for each 1 percent rise in total spending.

[47] This cannot explain modest effects of interest rates on willingness to pay for a home in the survey experiment of Fuster and Zafar (2021), where optimization frictions are unlikely to constrain behavior.

wealth and preference for homeownership is

$$\Pr(\kappa = \infty \mid w, \phi) = \Phi(a + b\left[\log \phi - \log \bar{\phi}(w)\right])$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and $\bar{\phi}(w)$ is the renting-buying unassisted indifference threshold for an agent with wealth w.

Under these assumptions, the joint distribution of wealth, preference for homeownership, and hassle cost is indexed by seven parameters, which we estimate using nine independent moments by classical minimum distance.[48] Our estimation targets the marginal buyer, inframarginal taker, buyer never-taker, and renter never-taker shares reported in Appendix Table A.3; the down payment share distribution among inframarginal takers when buying unassisted; and the down payment share distribution of buyer never-takers.[49] We restrict the median wealth of marginal buyers to be no less than the median wealth of the least wealthy third of inframarginal takers. Conditional on calibrated parameters, down payments depend only on wealth, so inframarginal takers' down payment shares when buying unassisted help identify the wealth distribution. The distribution of down payments among buyer never-takers helps identify the distribution of the hassle cost by showing how take-up declines with wealth. The type shares inform the distribution of preference for homeownership and its correlation with wealth, since the share of agents who buy unassisted depends on how many have preferences above the threshold $\bar{\phi}(w)$ at their wealth. The wealth restriction is necessary because we do not observe the down payments that marginal buyers would make if forced to buy unassisted, nor do we have any direct information about their wealth. We weight each moment by the inverse of its estimated sampling variance so that precisely estimated moments receive more weight in the criterion.

Panel C of Table 2 shows estimation results alongside the calibration choices. Figure 11 shows that the model-generated moments are within 95% confidence intervals of all but one of the empirical moments (the 10–20% down payment bin among inframarginal takers), and most closely match the more precisely estimated moments, namely, the type shares and inframarginal takers' down payments.[50] The wealth restriction binds at the estimates: me-

[48] The type shares and down payment distributions have 12 moments in total; however, only nine are independent because each block of shares sums to one.

[49] The targeted marginal buyer share, 43.9 percent, differs slightly from the regression-adjusted effect of a Dream for All offer on homeownership reported in Section 4 (42.5 percentage points) because the trinomial estimates weight regions by their shares of applicants rather than by the within-region variance of offer status, and count as Dream for All buyers 11 treated applicants whose Dream for All loan appears in CalHFA records but who have no open mortgage in credit records in the fourth quarter of 2025.

[50] Appendix Figure A.9 shows that, for every parameter except the level $\Phi(a)$ of the hassle cost, the criterion attains a stark minimum at the estimated parameter value. Because this hassle cost parameter is not sharply identified, we probe the sensitivity of our estimates to different values.

dian wealth among marginal buyers equals that of the least wealthy third of inframarginal takers. The overidentification test rejects the model, with $J = 11.9$ on 3 degrees of freedom ($p = 0.008$).[51] The rejection reflects the precision of the targeted moments rather than a large misfit: the model misses one of the eleven moments in Figure 11, the share of inframarginal takers putting 10–20% down when buying unassisted, by 2 percentage points, and the remaining ten by less than 1.5 standard errors. Our estimates imply that a one standard deviation increase in preference for homeownership ϕ conditional on wealth (i.e., $\sigma_\phi\sqrt{1-\varrho^2}$), starting from the median, raises the likelihood of unassisted homebuying from 32% to 70%. The estimates also imply the first, second, and third wealth quartiles are 6.0%, 23.5%, and 92.0% of applicants' median home purchase price (including Dream for All-assisted purchases). Setting the scale of the model so the average price paid by an unassisted inframarginal buyer is \$640K (Table 1), a \$50K increase in wealth for a median-wealth agent increases her likelihood of unassisted buying from 32% to 57% (25 p.p.). This implies that the modeled agents' housing tenure choices are highly sensitive to liquidity: for comparison, Bulman, Goodman and Isen (2023) find that a lottery win between \$50K and \$100K (\$72K on average) increases homeownership by 6 percentage points among non-owners. The strong model-implied liquidity effect likely reflects selection into applying for Dream for All, which required applicants to secure contingent mortgage pre-approvals and complete homebuyer education, so that the only barrier for homebuying in the modeled (and experimental) population was liquidity. Declining the offer due to hassle cost almost exclusively occurs among agents who can buy unassisted: the estimated share of these applicants who decline is 33%, 32%, 33%, and 27% across the four wealth quartiles, but is about 1% among marginal buyers. The parameter estimates also imply that 35% of marginal buyers are deterred from buying unassisted by the minimum down payment.

Given calibrated and estimated parameters we solve numerically for applicants' WTP for the Dream for All loan. We calculate WTP as equivalent variation (EV): for an agent who prefers to buy with Dream for All, how much wealth could we take away from them in this preferred alternative before they no longer prefer it to their fallback option (renting for marginal buyers, buying unassisted for inframarginal takers).[52] Expected fiscal cost per dollar of principal is the product of the shortfall of Dream for All's appreciation-sharing

[51] The degrees of freedom calculation takes nine independent moments minus seven parameters, plus one degree of freedom for the binding wealth restriction.

[52] Using EV to measure WTP is a conservative choice because it bounds willingness to pay at an agent's full wealth endowment. An alternative approach could use compensating variation, asking how much we would have to compensate applicants who prefer Dream for All in their fallback option to make them indifferent with their preferred Dream for All allocation. Using this alternative measure dramatically increases WTP for Dream for All among poor applicants who strongly prefer to own, as shown in Appendix Figure A.10 Panel B.

parameter below the actuarially fair one, $\gamma_{\text{fair}} - 1$, and the expected present value of appreciation per dollar of principal, $\pi g^{+}/(1+\rho)$, which is what one unit of appreciation sharing is worth to the state.

7.2 Baseline estimates and sensitivity analysis

Figure 12 Panel A.I shows WTP for Dream for All and its fiscal cost under our baseline assumptions, normalized by the total number of Dream for All dollars deployed. Average WTP was 7.1 cents per dollar, which greatly exceeded Dream for All's average cost of 1.3 cents. Taking the ratio of the two, Dream for All has an MVPF of 5.6 (Panel A.II). Recall that a $1 lump-sum transfer has an MVPF of 1: recipients value it at $1 and it costs the government $1 to provide. WTP for Dream for All is much lower because Dream for All recipients had to pay back the government: the transfer is merely the difference between the cost of Dream for All-provided credit and the credit available in the private market. Also, marginal buyers had to change their housing tenure and, correspondingly, housing and non-housing consumption, to benefit from the subsidized financing.

This average WTP figure masks heterogeneity across applicants. Appendix Figure A.10 Panel A shows how WTP varies across the type space diagram presented in Figure 4. WTP is highest among relatively low-wealth inframarginal takers: inframarginal takers whose wealth is between 20% and 100% of their income are willing to pay over 15 cents per dollar of Dream for All, over twice the population-wide average. WTP is also high among marginal buyers very close to the margin of unassisted buying, and declines steeply further from the indifference threshold. This decline occurs for very different reasons on either side of the curve: to the right, as agents' wealth grows, cost savings and risk-smoothing from Dream for All are not especially valuable because agents are wealthy enough that they comfortably make cost-minimizing cash down payments and can self-insure fairly effectively against home price risk. Down and to the left of the renting-unassisted buying indifference curve, WTP is lower at first because benefitting from Dream for All requires switching to homeownership when they are almost indifferent between owning and renting. Further up and to the left, in the region where applicants have very high preference for homeownership and very low wealth, relatively low WTP hinges instead on our use of EV rather than compensating variation: many of these applicants would sacrifice all of their wealth to buy with Dream for All rather than renting, but their wealth is not very large. Appendix Figure A.10 Panel B shows that WTP in this population is much higher when we measure WTP instead as compensating variation.

Given that WTP is highest among constrained inframarginal takers, it is perhaps not

surprising that the moment in the data most pivotal for our baseline WTP estimate is the share of inframarginal takers who would make cash down payments below 5% of the home purchase price. Appendix Figure A.11 Panel A shows average WTP per dollar in the applicant sample when estimating the model at (counterfactual) alternative target moments, varying the share of inframarginal takers making down payments below 5% between 10% and 70%. If only 10% of inframarginal takers made down payments below 5%, average willingness to pay for Dream for All would fall to 5 cents per dollar deployed; if, instead, 70% of inframarginal takers made sub-5% down payments, average WTP would rise to 10 cents per dollar. Panel B shows, in contrast, that average WTP varies little when we estimate the model to match counterfactual marginal buyer population shares. As discussed above, the fact that the modal inframarginal taker makes a down payment under 5% when buying unassisted implies that their consumption is scarce: they are willing to pay high interest rates (and, in the real world, mortgage insurance premiums and fees) to preserve what little liquidity they have in the period in which they buy homes. However, they are far enough from indifference between homeownership and renting, even without assistance, that they willingly sacrifice consumption in period 0 and, because of high mortgage costs, in period 1 to be homeowners. Dream for All offers these agents relief from their period-0 consumption strain without requiring them to give up the homeownership they so value.

This discussion implies that our model will give incorrect estimates of WTP for Dream for All if it imposes an unrealistic mapping between period-0 consumption scarcity and the down payment share of an unassisted buyer. We use observational data from the 1995–2022 waves of the Survey of Consumer Finances to probe the model's relationship. We identify survey participants who bought homes for the first time in the survey year or the year before, and track the home purchase prices and down payments that they made. We then rank these first-time buyers by their non-housing wealth, adding back in the cash down payments they made, and compare the relationship observed in the data to model-implied down payment choices. To make the wealth scale of the data comparable to that of the model, we group buyers into twenty wealth bins and divide median wealth by median home purchase price in each bin. Appendix Figure A.3 shows the relationship between wealth, purchase price, and down payment share in our model and in the SCF data. The vertical axis shows the share of a home purchase made in cash, and the horizontal axis shows wealth normalized by (unassisted) home purchase price. Reassuringly, the model-implied relationship between wealth and down payment share closely fits these non-targeted moments. The median down payment share of a homebuyer in the 25th–30th percentile bin of the wealth distribution is 1.6%, and the average is 8.2%, compared to 3.5% in the model. The model replicates the increase in down payment share for buyers higher in the non-housing wealth distribution, as

well as the plateau in down payments at 20% for buyers whose non-housing wealth is greater than half their home's purchase price.

Appendix Figure A.12 shows how estimated WTP and fiscal costs (Panel A) and the MVPF (Panel B) vary when we adopt alternative assumptions around the home price process, household preferences, the financial environment, government discounting, and recovery of principal in the low home price state. Panel A shows that, across scenarios increasing and decreasing the likelihood of the low home price state by 100% and 25%, respectively; increasing and decreasing the magnitude of appreciation by 25% and 50%, respectively; and increasing and decreasing the magnitude of depreciation by 50%, WTP for Dream for All and fiscal cost move less than 2 cents per dollar of principal. The home price scenario that moves WTP the most is a smaller decrease in home prices, which decreases WTP by 1.5 cents per dollar, as Dream for All is most valuable when home prices fall (in which case it is a zero-interest loan). Fiscal cost increases the most if upside is halved, since the government earns when home prices go up.

Halving the price of the subsistence home reduces WTP for Dream for All by about 1.3 cents per dollar of principal, because the consumption relief Dream for All provides is worth less when minimum housing expenditure is lower. Neither a fivefold increase nor a halving of the weight on housing β in utility significantly affects WTP for Dream for All. Halving or doubling the slope of the assumed mortgage rate schedule (Appendix Figure A.8) changes WTP by about a cent per dollar of principal or less. A halving of the government bond rate, the household discount rate, and the mortgage rate schedule increases WTP slightly because households discount the period-1 part of Dream for All's benefit less heavily, but makes Dream for All self-financing: the state's expected share of appreciation, 3.7% of principal per year, exceeds its 2.5% cost of capital. Dream for All is also self-financing under our baseline parametrization when we use a more realistic 3% discount rate, rather than the assumed 5%, to discount Dream for All cash flows. Under the other extreme assumption, where the government is no longer risk-neutral but has the same relative valuations of a dollar in the low and high home price states as the average Dream for All taker, the corresponding rise in fiscal cost is proportionally large (69%) but small relative to WTP.

Dream for All's expected fiscal cost grows more quickly when we relax the assumption that the state recovers the entire Dream for All principal when homebuyers are underwater. The figure shows that WTP still exceeds expected fiscal cost if 5-10% of principal is lost, but the two are roughly equal when a quarter of principal is lost, and cost is nearly twice WTP when half is lost. For reference, Davis, Larson, Oliner and Smith (2022) estimate that 16-18% of conventional loans with CLTVs above 96% (credit scores 720–769, DTI 39–50%) would default if a financial crisis of the magnitude of the Great Financial Crisis occurred shortly

after mortgage origination, given the national average price decline. Because Dream for All loans are junior to the underlying first mortgage, the state would likely recover nothing in these defaults. Taking estimates from the model and the literature seriously, defaults in a serious downturn could substantially reduce Dream for All's cost-effectiveness, as measured by the MVPF.

7.3 Alternative policies

Hendren and Sprung-Keyser (2020) suggest comparison of policies targeting a given population and using the MVPF to ascertain which deliver the highest "bang for buck." In this section, we compare WTP for Dream for All to a prominent transfer studied by Berger, Turner and Zwick (2020): the First-Time Homebuyer Credit (FTHC). The FTHC was enacted in 2008 and, in its most generous form (2009–2010), provided a refundable tax credit of 10% of the purchase price up to $8,000 to homebuyers who had not owned a principal residence in the preceding three years, with the credit phasing out at incomes above $75,000 for single filers ($150,000 for married filers). Given our model estimates, we simulate the effects of an FTHC for the Dream for All applicant population. We simulate the FTHC as a purchase-contingent transfer of $12K, fixing the model scale based on the median purchase price among applicant homebuyers and adjusting the $8,000 credit for inflation; the transfer is 2% of the median purchase price. Figure 12 Panel B shows the results. Panel B.I shows that WTP for Dream for All is much lower than for an FTHC, for which applicants would pay 69 cents per dollar of tax credit. Because the FTHC was a transfer, each dollar of assistance deployed through the program cost a full dollar, implying that the program's MVPF fell below one.[53] Despite much lower WTP, the Dream for All program beats the FTHC on MVPF because it recoups most of its cost.

This policy comparison can be justifiably characterized as unfair to the FTHC for two reasons. First, the FTHC's MVPF falling below one is not a quantitative question: any behavior-contingent transfer will have an MVPF less than one, before accounting for fiscal externalities. Second, this exercise compares the two policies on utilitarian terms: a policy is worth only what households would pay for it, and homeownership has no value apart from those households' private valuations. In practice, policymakers promoting assistance

[53]This assumes away fiscal externalities. Homeownership could generate positive fiscal externalities through increased property tax payments or causal effects on earnings (e.g., the responsibility of a mortgage causes people to work more). It could also generate negative fiscal externalities through deductibility of mortgage interest or, as suggested by Figure 10, mobility to areas with lower earnings opportunities or effects on children. Regardless of the net direction, we see no reason to think that fiscal externalities from FTHC-induced increases in homeownership would differ from Dream for All-induced increases in homeownership, so including them on a per-dollar basis would be unlikely to reverse our qualitative conclusions.

to homebuyers often argue along non-utilitarian lines, arguing that owner-occupied housing is a merit good whose consumption carries special benefits, or arguing that homeownership carries externalities which warrant corrective subsidies (Glaeser and Shapiro, 2003).[54]

To evaluate the policies on a more level playing field, we compare each policy's homeownership gains in the applicant population to its fiscal costs. The FTHC generates larger homeownership gains in the applicant population, 54.4 percentage points, than Dream for All's 43.9. However, these gains come at a cost nearly an order of magnitude higher. Fixing the scale of the model again so that the average unassisted home purchase is $640K (Table 1), Dream for All's cost per induced homeowner is $2.3K, while the FTHC's cost is $18.3K per induced owner.[55]

8 Conclusion

Governments play important roles in setting the terms of mortgage markets. Historically, the emergence of new mortgage products in the United States has been driven not by private innovation, but by government support (Schwartz, 2021). Many housing finance scholars and policymakers, chastened by the consequences of loose lending in the Great Financial Crisis, are skeptical that governments should throw their considerable weight behind mortgage market innovation (e.g., Immergluck, 2011). However, this skepticism is in tension with the general principle that more complete financial markets can improve welfare, for example, by alleviating liquidity constraints (Shiller, 2014).

This paper shows that the limited-scale introduction of a government-supported shared appreciation loan dramatically increased homeownership not by setting lower standards for mortgage qualification, but by straining liquidity less than status quo financing. Our quantitative analysis paints a generally optimistic picture of the prospects for government innovation to improve homebuyer welfare.

We note that our study provides little direct evidence on whether the apparent benefits of shared appreciation loan financing are likely to generalize beyond the Dream for All setting. Expanding shared appreciation loans at a larger scale would increase demand for owner-occupied housing, which in turn could raise home prices. These general equilibrium effects would likely erode shared appreciation loans' effectiveness in facilitating transitions

[54] President Clinton introduced the National Homeownership Strategy by claiming that "[w]hen we boost the number of homeowners in our country, we strengthen our economy, create jobs, build up the middle class, and build better citizens" (Clinton, 1995); President Bush defended his homeownership agenda on the grounds that "if you own something, you have a vital stake in the future of our country" (Bush, 2004).

[55] About a third of both policies' spending goes to inframarginal takers, so the cost per induced owner gap reflects a gap in per-beneficiary cost: a Dream for All loan costs the state $1.6K in expectation, while every credit claimant costs $12K.

from renting to homeownership.[56] Implications for the broader housing market could be more nuanced. If markets for rental and owner-occupied housing are segmented, price increases in the owner-occupied sector could be accompanied by short- or medium-term price reductions in the rental sector as landlords are slow to convert rental properties to owner-occupiable units (Greenwald and Guren, 2025).[57] Ultimately, the dynamics of a broader shared appreciation loan expansion are beyond the scope of our study, to be assessed more rigorously in the context of dynamic general equilibrium models. Future work can use the rich experimental evidence provided here to discipline its models, providing more credible answers to more general questions.

[56] Ochoa (2025) shows empirically that a Chilean homebuyer subsidy increased home prices, and that targeting assistance to marginal homebuyers increases subsidies' price effects. Sommer and Sullivan (2018) and Coven, Golder, Gupta and Ndiaye (2024) use quantitative models to show that tax subsidies for homeownership put first-time homebuyers at a disadvantage through price effects.

[57] Kline and Walters (2016) similarly find that Head Start can generate indirect welfare gains by reducing congestion in competing preschools, increasing the total number of children who can attend some preschool.

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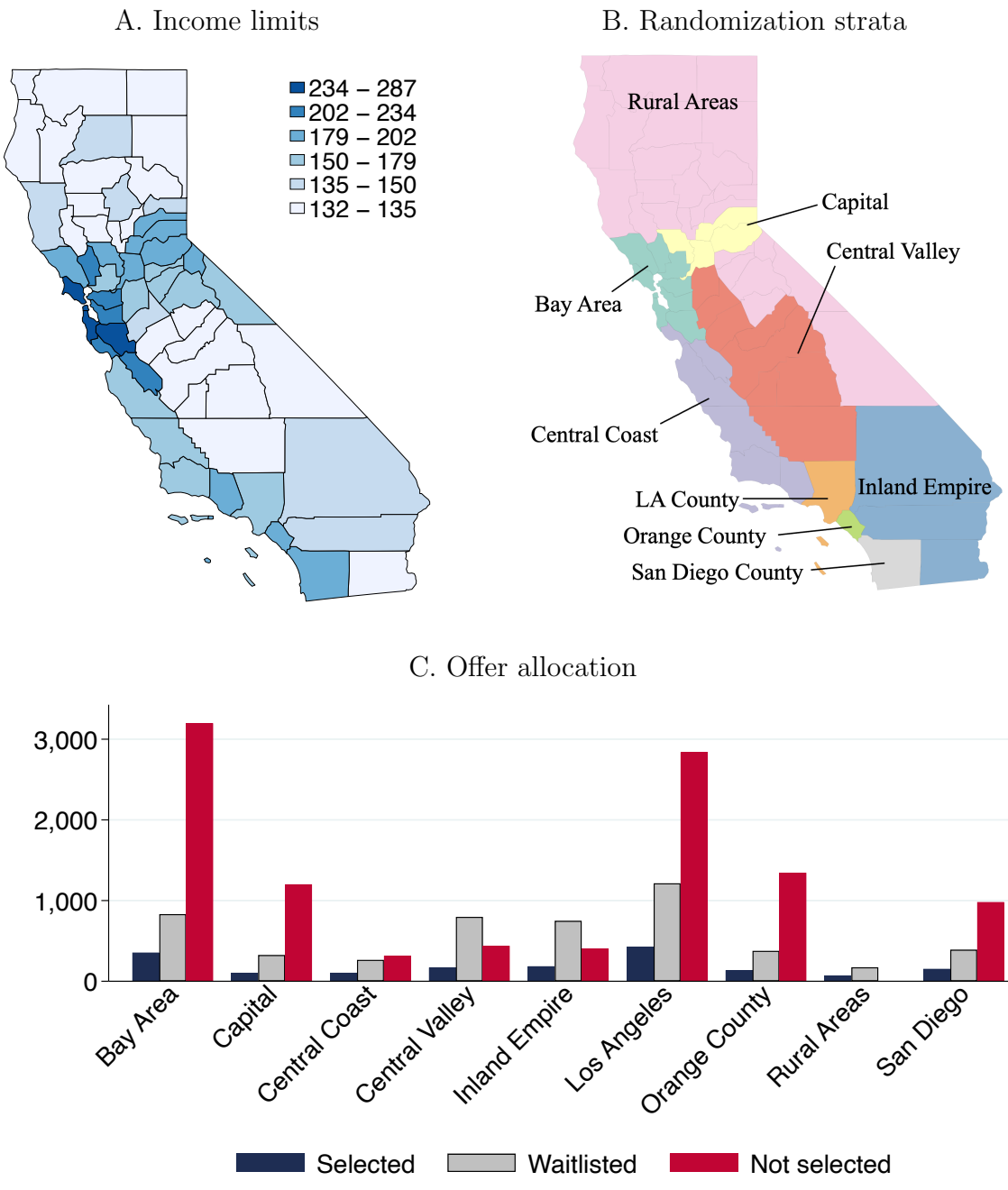
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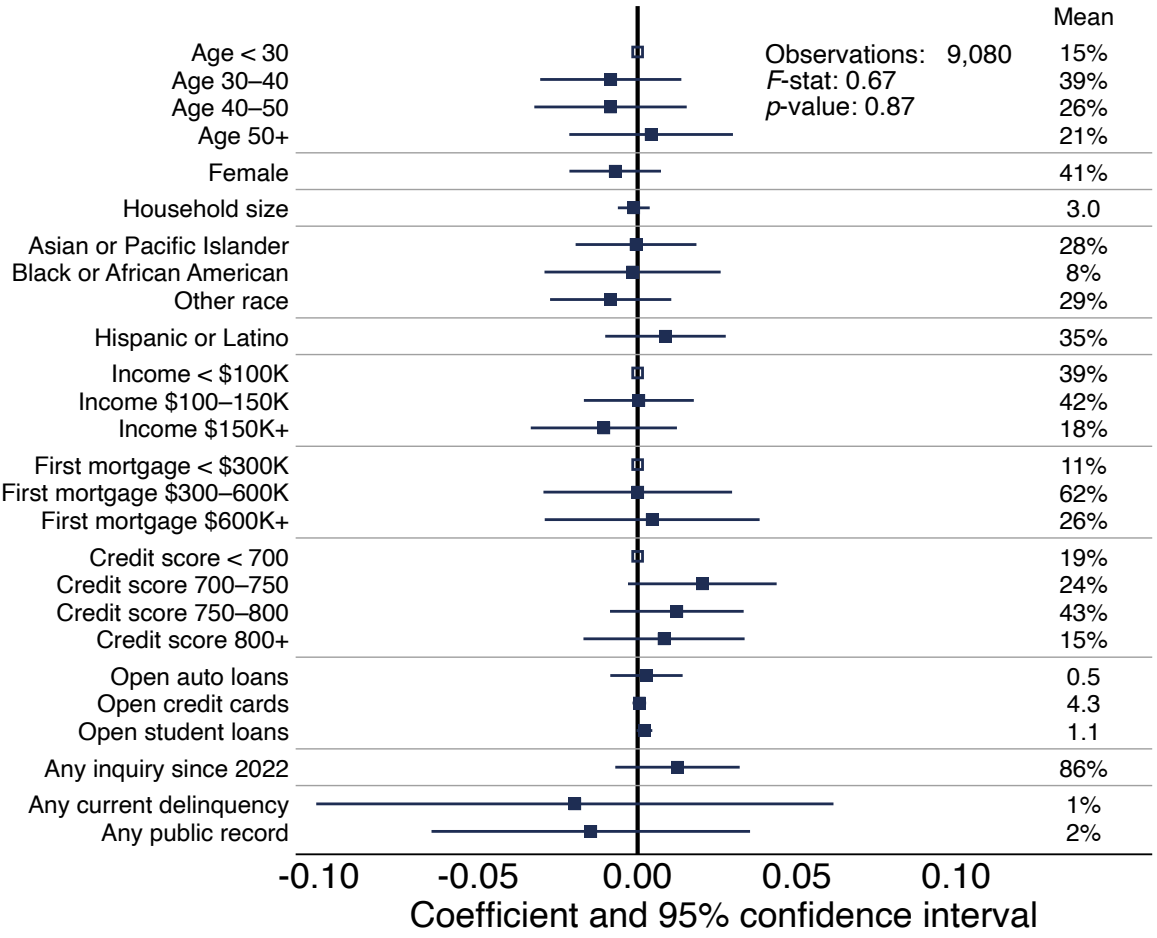
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Figure 1: Dream for All geographic program parameters



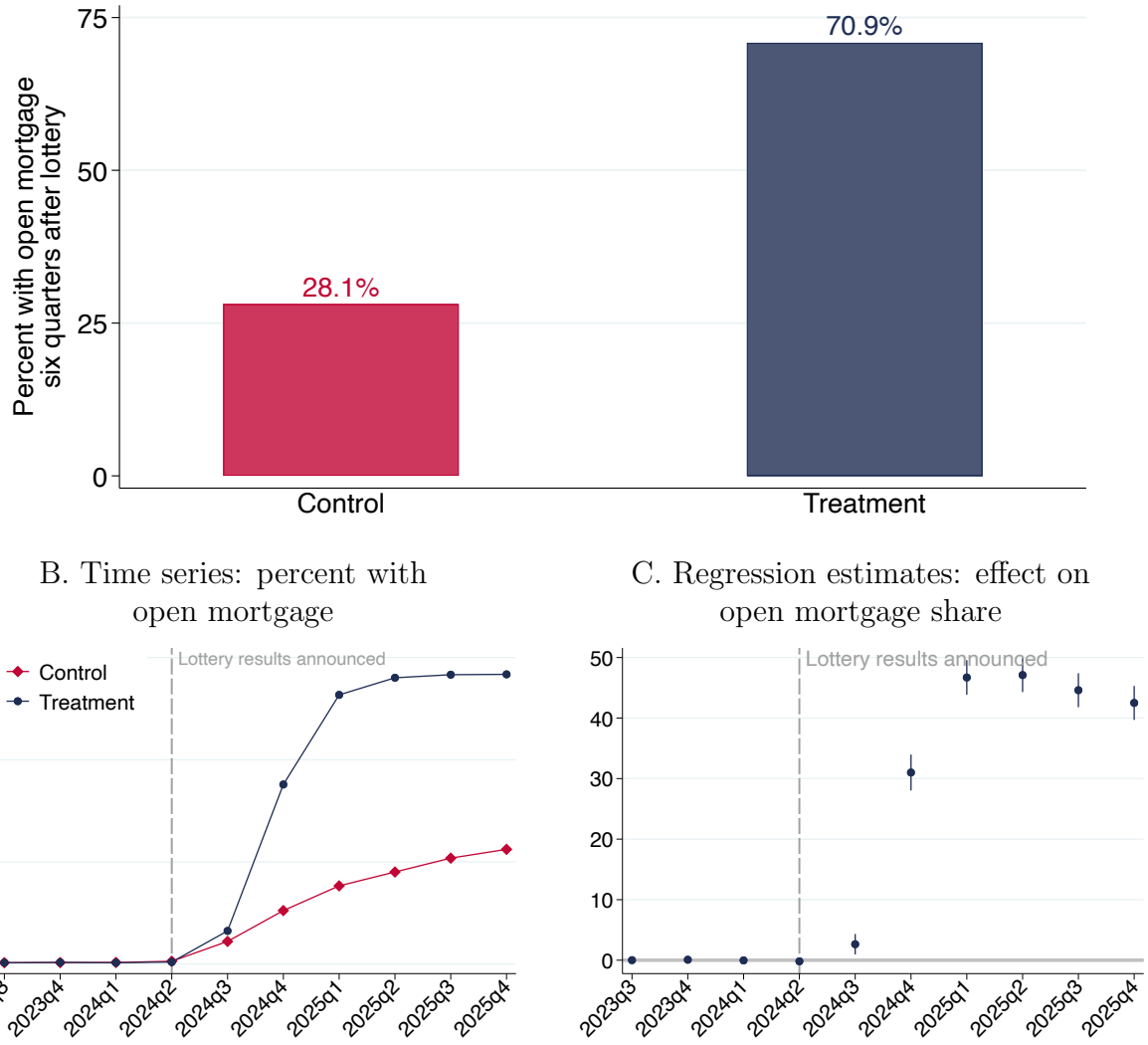
Notes: This figure shows the geographic parameters of the Dream for All program: the county-specific income limits applicants had to earn below, the regions used as randomization strata in the Phase 2 lottery, and how lottery outcomes were allocated across regions. Panel A shows county-level income limits (in thousands of dollars) for the Dream for All program, corresponding to 120% of the Department of Housing and Urban Development's Area Median Income index for a family of 8. Panel B shows the nine geographic regions (groups of counties) used as randomization strata. Panel C shows the allocation of applicants across lottery outcomes. Income limit data (June 2024) and regional definitions are from the California Housing Finance Agency.

Figure 2: Covariate balance and summary statistics



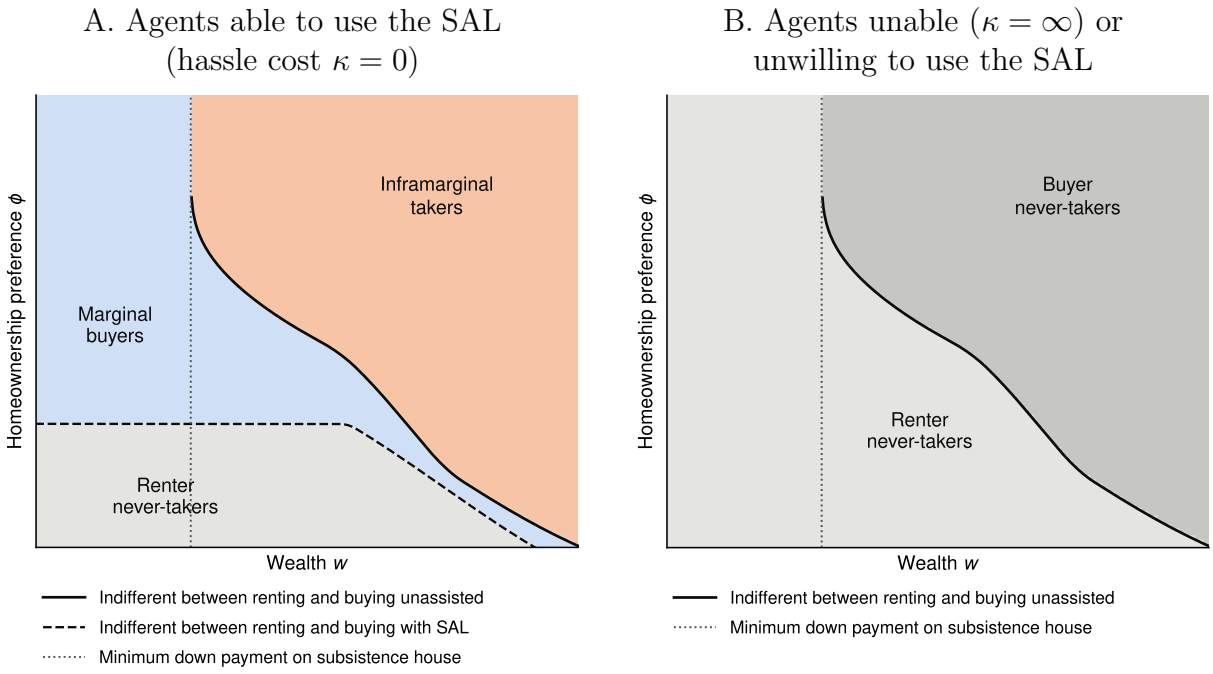
Notes: This figure tests for covariate balance between the treatment and control groups and summarizes the characteristics of the analysis sample. It plots coefficients from a single regression of an indicator for being initially offered a Dream for All loan on the covariates listed at left, controlling for region fixed effects. Squares show point estimates and horizontal bars show 95% confidence intervals based on heteroskedasticity-robust standard errors. Hollow squares mark omitted reference categories. The right-hand column reports the sample mean of each covariate, adjusted for region: each covariate is regressed on region indicators and the resulting region-specific means are averaged with equal weight across the eight regions in the analysis sample. Means are shown as percentages for indicator variables and as levels otherwise. The box reports the number of applications, the F-statistic, and the p-value for the joint null that all coefficients other than the region fixed effects are zero. Age, gender, household size, race, and ethnicity are from Dream for All pre-registration applications; total credit qualifying income sums income across all co-borrowers on an application; first mortgage pre-approval amounts are from CalHFA-approved lenders; credit score, open accounts, inquiries, delinquencies, and public records are from the UCCCP as of 2024Q1. The sample is the analysis sample defined in Section 3.

Figure 3: A Dream for All offer dramatically increases homeownership



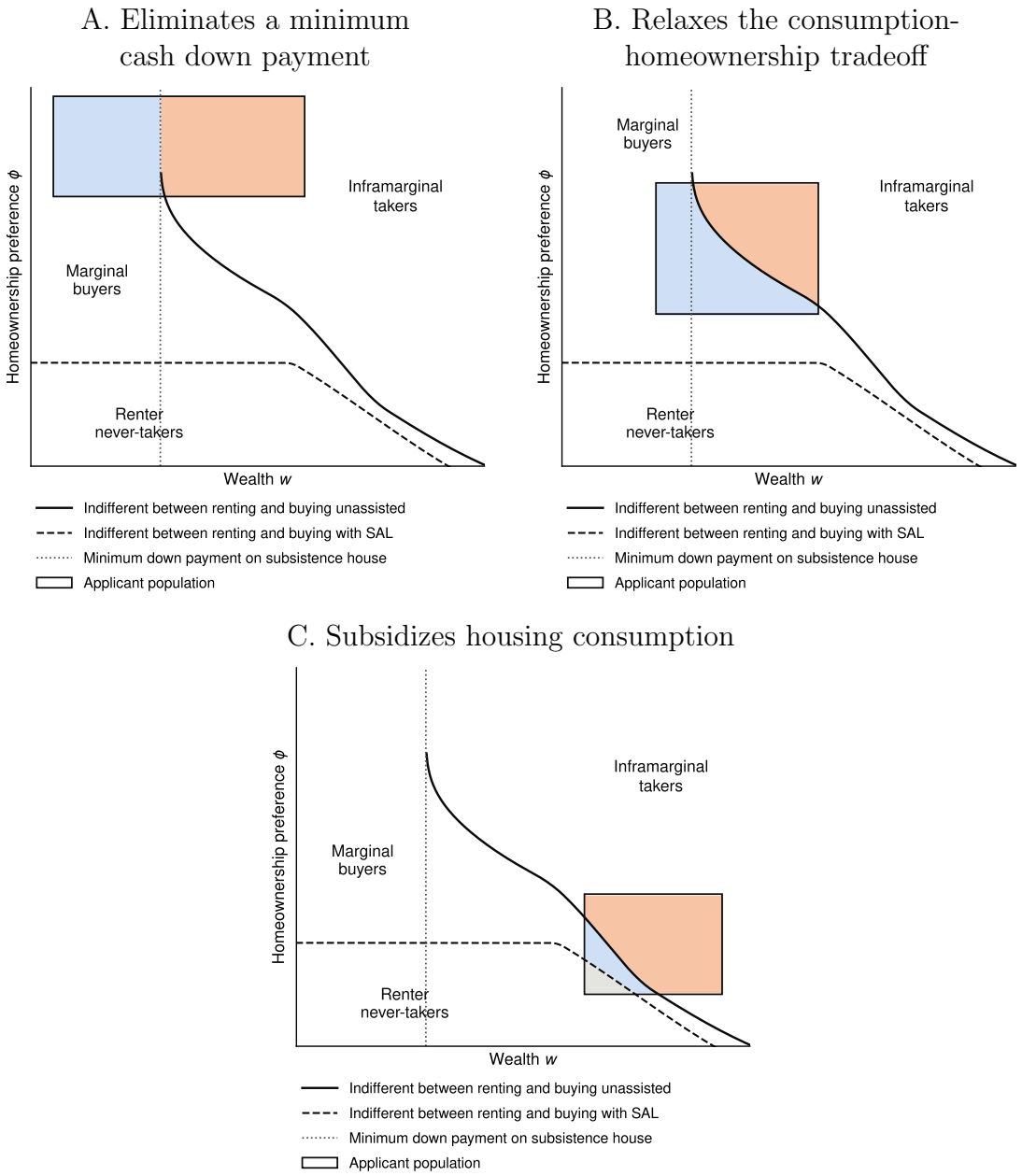
Notes: This figure shows the causal effect of access to a Dream for All loan on homeownership, and the evolution of the effect over time. Panel A shows the share of the treatment group (applicants initially offered a Dream for All loan) and the control group (applicants not selected) with an open mortgage six quarters after the Dream for All lottery. Panel B shows the share of the treatment and control group with open mortgages in each quarter from the third quarter of 2023 to the fourth quarter of 2025. Panel C shows the corresponding causal effect of a Dream for All offer at each time horizon, measured as the coefficient β from the regression $\text{BUY}_i = \alpha + \beta\text{LOTTERY}_i + \mathbf{R}_i\Gamma + \varepsilon_i$, where BUY_i is an indicator for having an open mortgage at a given time horizon, LOTTERY_i is an indicator for receiving a Dream for All lottery offer, and $\mathbf{R}_i$ is a vector of randomization strata. Vertical bars show 95% confidence intervals based on heteroskedasticity-robust standard errors. The dashed vertical line in Panels B and C marks the quarter in which lottery results were announced. The sample is the analysis sample described in Section 3.

Figure 4: A partition of the applicant population



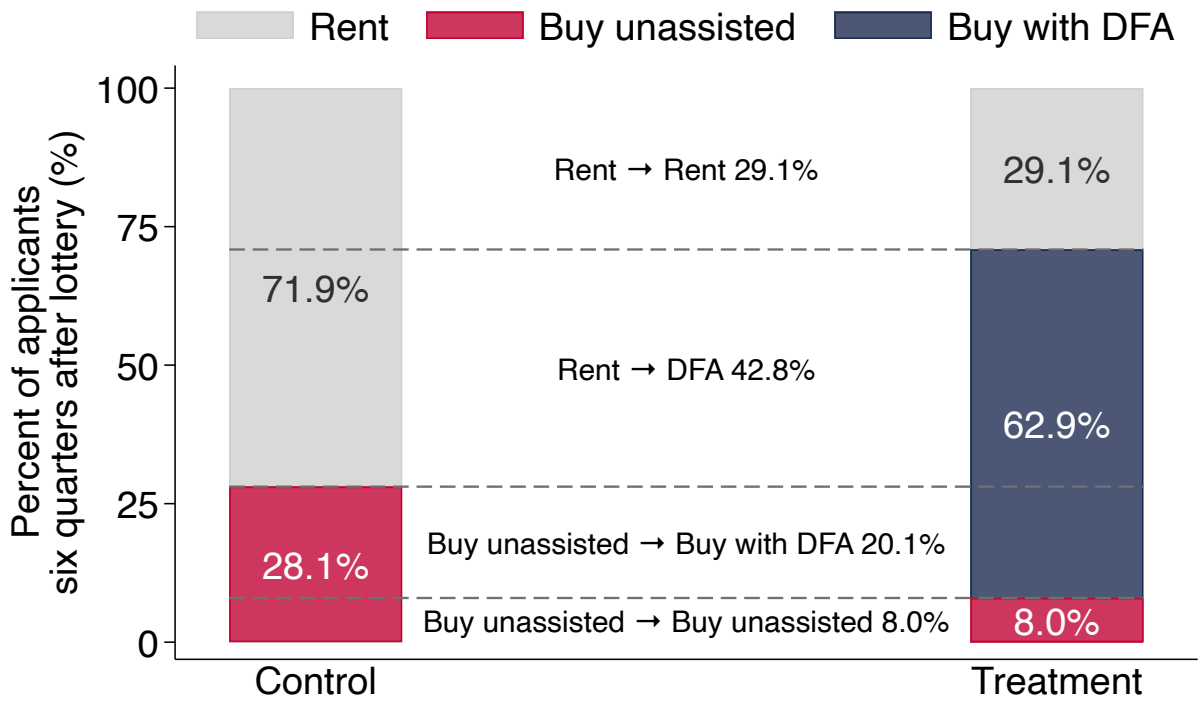
Notes: This figure shows how the model divides the applicant population into four types (marginal buyers, inframarginal takers, renter never-takers, and buyer never-takers) according to their rankings of renting, buying unassisted, and buying with the SAL. Each panel partitions agents by wealth w (horizontal axis) and preference for homeownership ϕ (vertical axis). The solid line is the minimum preference for homeownership at which an agent prefers buying unassisted to renting; the dashed line is the minimum preference at which an agent prefers buying with the SAL to renting; the dotted vertical line is the minimum down payment on a subsistence house, to the left of which unassisted purchase is infeasible. Panel A shows agents who can use the SAL at no hassle cost. Marginal buyers (blue) prefer buying with the SAL to renting and renting to buying unassisted, either because they cannot afford the minimum down payment (left of the dotted line) or because they are below the solid line; inframarginal takers (orange) prefer buying with the SAL to buying unassisted to renting; renter never-takers (grey) prefer renting to both forms of ownership. Panel B shows agents whose hassle cost precludes using the SAL. Buyer never-takers (dark grey) prefer buying unassisted to renting; renter never-takers (light grey) prefer renting to buying unassisted, including those who would be marginal buyers but for their hassle cost. Marginal buyers and inframarginal takers are compliers, who buy with the SAL when it is offered; the share of applicants who are marginal buyers equals the causal effect of access to an SAL on homeownership, since it is the share induced to become homeowners by SAL availability.

Figure 5: Why did Dream for All increase homeownership? Three scenarios



Notes: This figure shows the model's implications for judging which feature of Dream for All explains its causal effect on homeownership. In Panel A, all marginal buyers have wealth below the minimum down payment, so every marginal buyer is poorer than every inframarginal taker, and we should see little overlap in the two groups' distributions of characteristics correlated with wealth. In Panel B, marginal buyers and inframarginal takers overlap in wealth, and both have wealth close enough to the minimum down payment that unassisted purchase strains period-0 consumption, so inframarginal takers with the same observable resources as marginal buyers make down payments below the cost-minimizing share. In Panel C, both groups are wealthy enough that the period-0 budget constraint does not bind, and the SAL is taken up only because it lowers the cost of owner-occupied housing, so inframarginal takers make cost-minimizing cash down payments and consume strictly more housing with the SAL than without it. Section 6 distinguishes between these scenarios empirically.

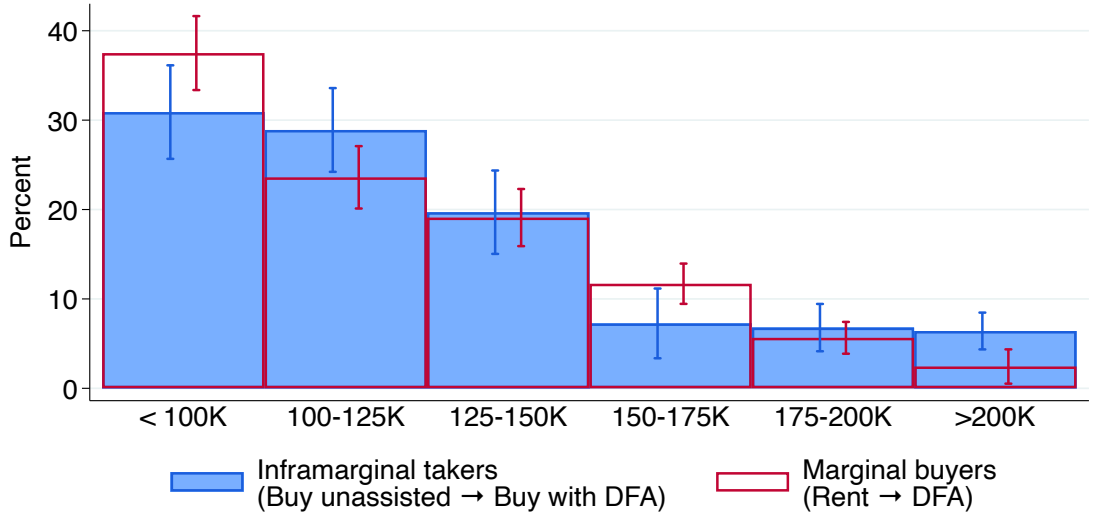
Figure 6: Identifying population shares of applicant types



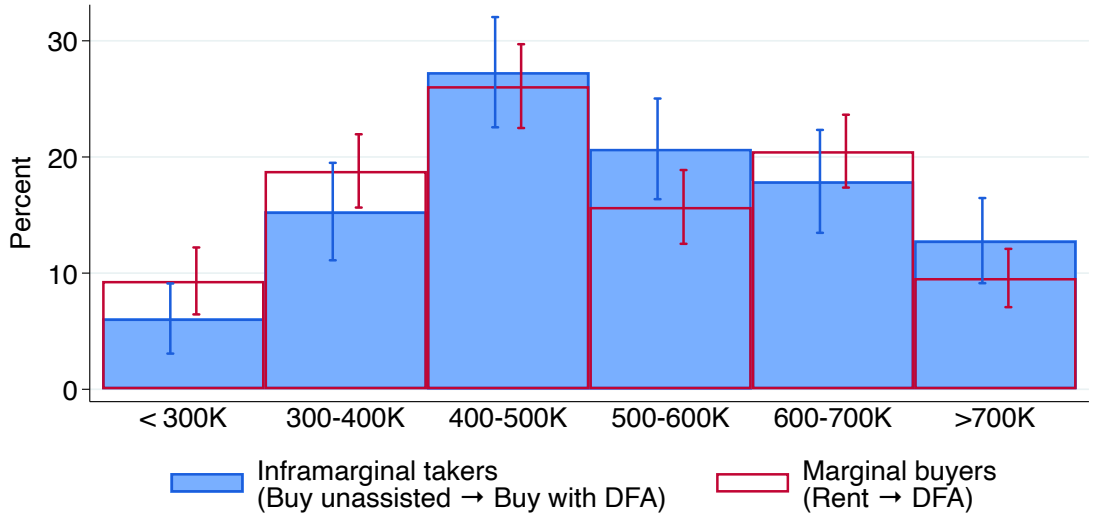
Notes: This figure shows how the random lottery identifies the applicant population shares of the four types defined in Section 5.2. Bars show the shares of control-group and treatment-group applicants who rent, buy unassisted, and buy with Dream for All six quarters after the lottery; homebuying is measured by an open mortgage in credit records, and the share buying with Dream for All is the program take-up rate among the treatment group. Dashed lines connect the bars at the control-group homebuying share, the treatment-group homebuying share, and the treatment-group unassisted-buying share. Labels between the bars give the share of applicants making each transition from control-group to treatment-group choice, which identify the four applicant types under monotonicity: renter never-takers (rent → rent) and buyer never-takers (buy unassisted → buy unassisted) are the treatment-group shares who rent and buy unassisted; marginal buyers (rent → DFA) are the treatment–control difference in homebuying; inframarginal takers (buy unassisted → buy with DFA) are the control-group unassisted-buying share less the buyer never-taker share.

Figure 7: Marginal buyers and inframarginal takers have similar incomes, pre-approvals

A. Total credit qualifying income

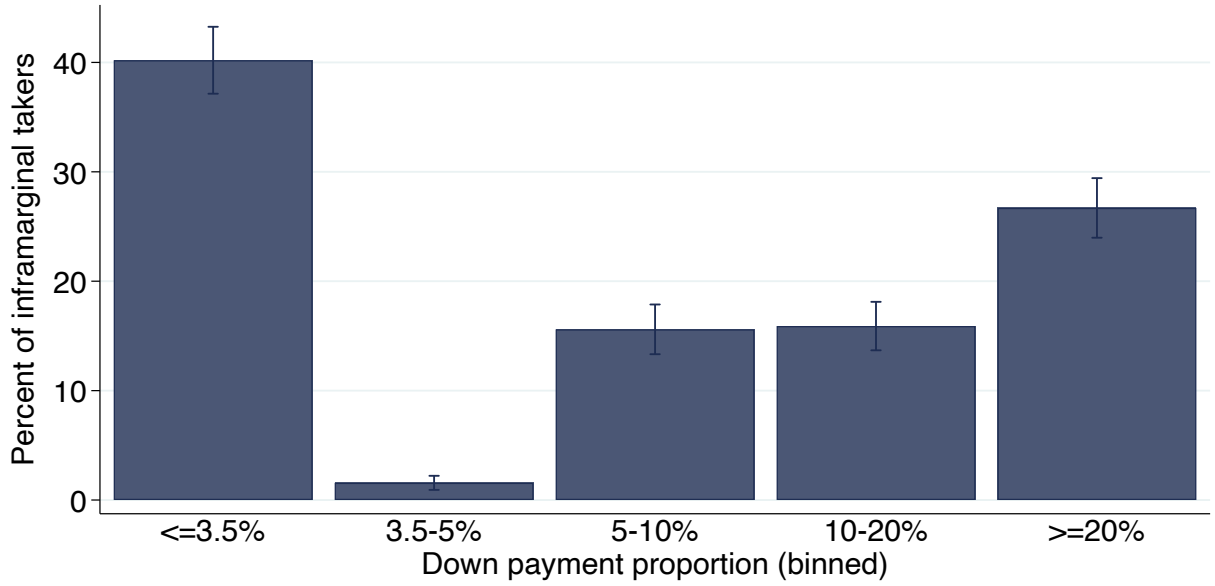


B. Mortgage pre-approval contingent on Dream for All offer



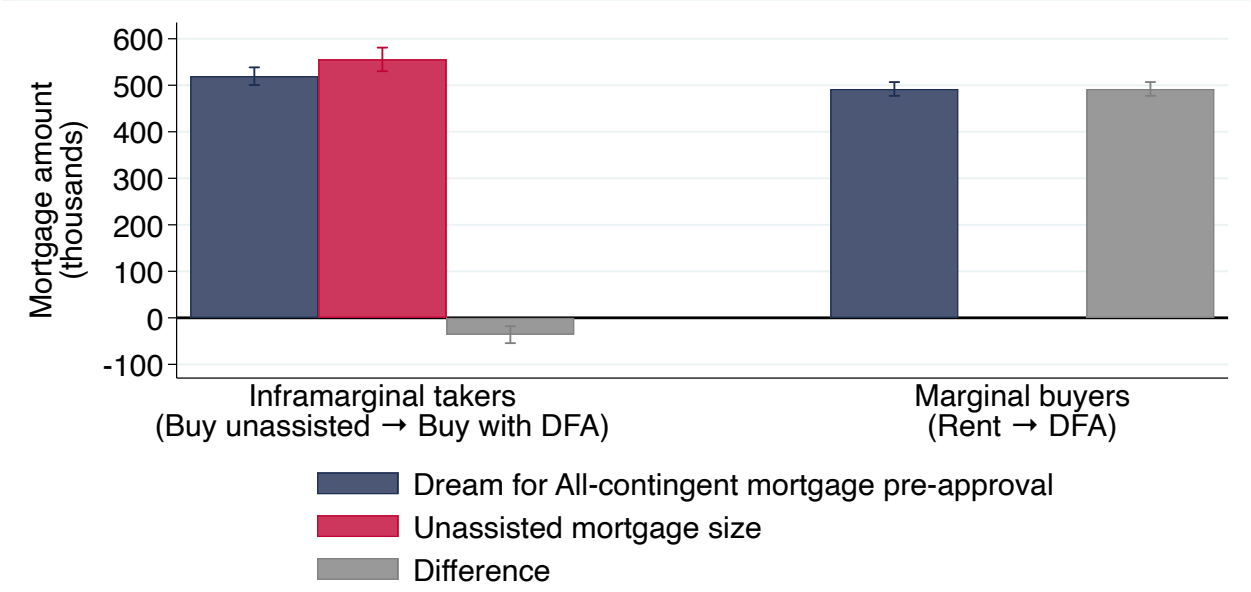
Notes: This figure tests whether elimination of the minimum cash down payment explains the causal effect of Dream for All on homeownership, which would require every marginal buyer to be poorer than every inframarginal taker (Figure 5 Panel A), by comparing the two groups' resources. Bars show the share of marginal buyers and inframarginal takers in each bin of total credit qualifying income (Panel A) and Dream for All-contingent mortgage pre-approval (Panel B), both recorded at application. Each bar is a two-stage least squares estimate from the regression system in equation 2, with the outcome set to an indicator for the bin; whiskers are 95% confidence intervals.

Figure 8: Inframarginal takers' down payment shares when buying unassisted



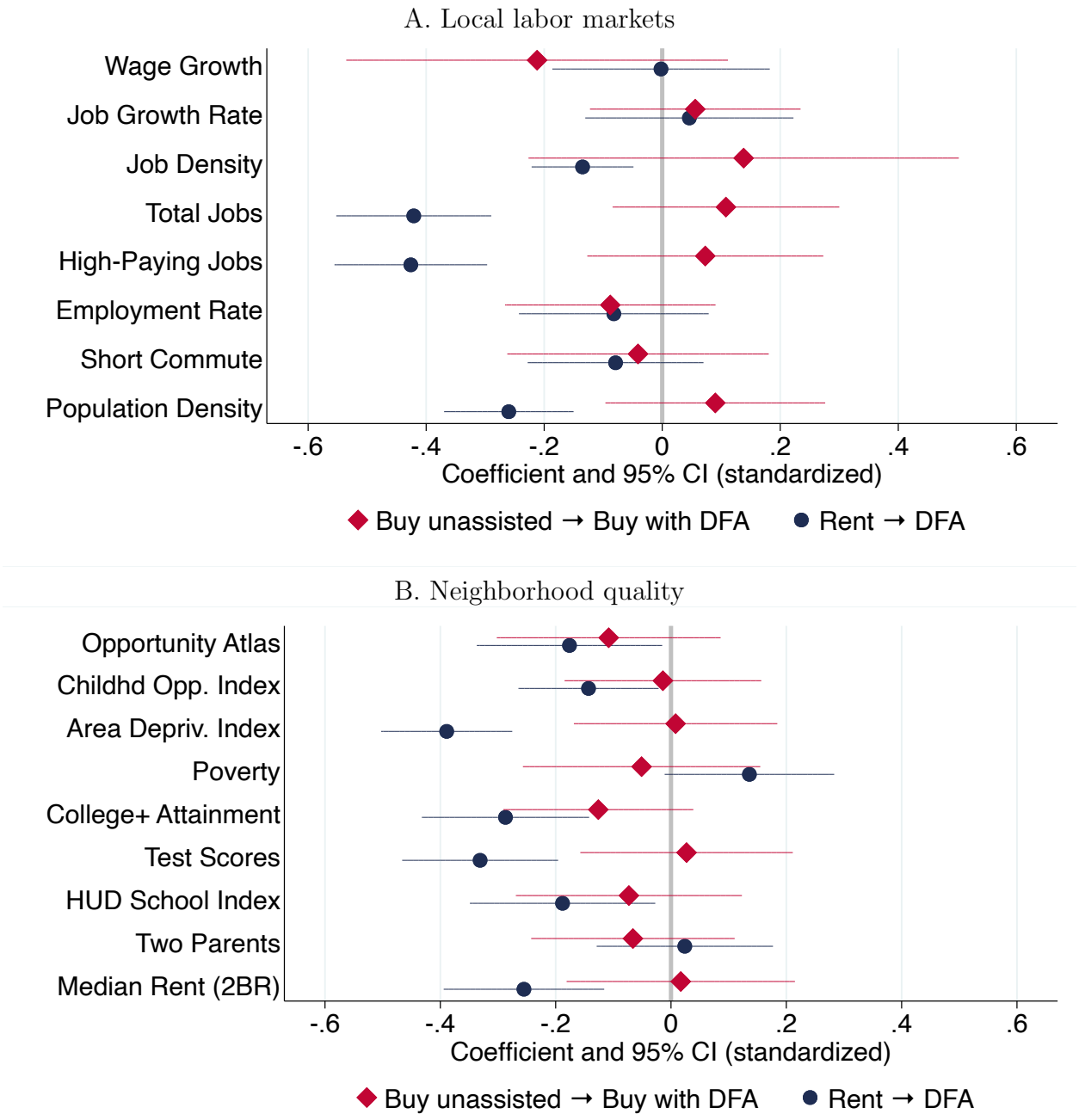
Notes: This figure shows the distribution of down payments that inframarginal takers, compliers who would buy without Dream for All, would make if they bought unassisted. Bars show the distribution of cash down payments as a share of the purchase price among inframarginal takers buying unassisted, estimated as a two-stage least squares estimate from the regression system in equation 2 with the outcome set to an indicator for each bin; whiskers are 95% confidence intervals.

Figure 9: Inframarginal takers' unassisted mortgages are larger than their Dream for All pre-approvals



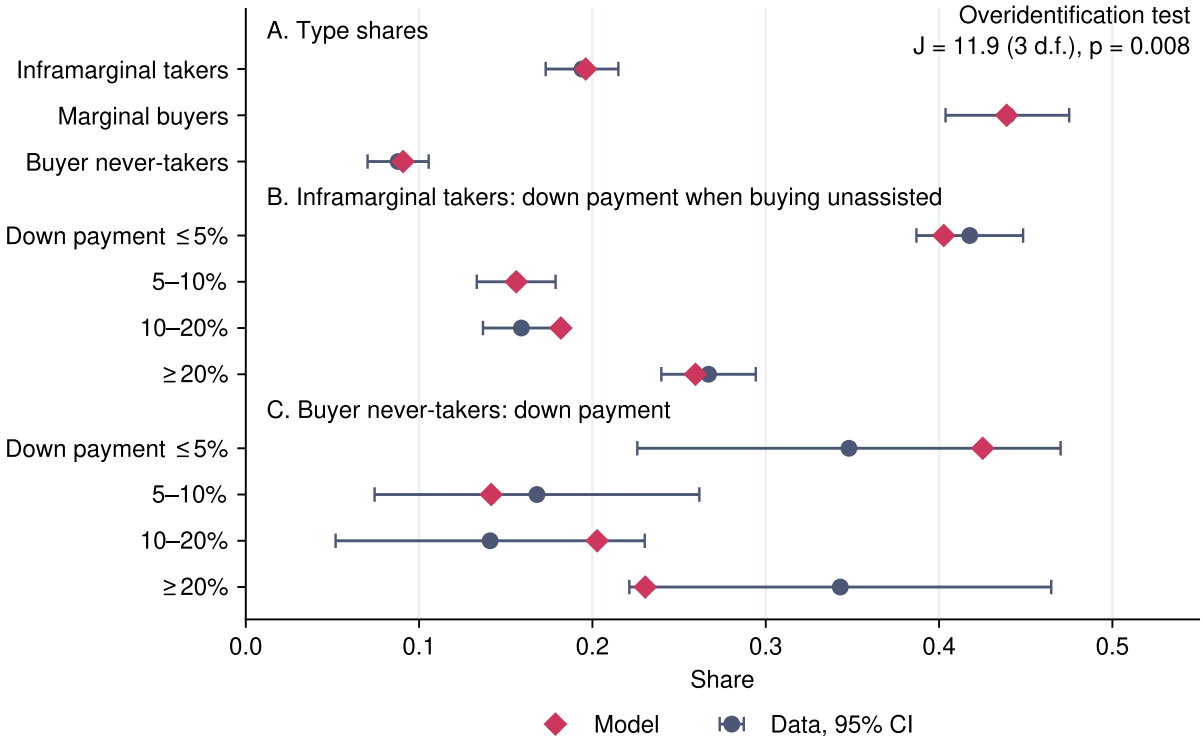
Notes: This figure tests whether applicants used Dream for All to increase housing consumption by increasing mortgage borrowing, by comparing inframarginal takers' mortgage size when they buy unassisted to their Dream for All-contingent pre-approvals. Bars show, for each complier group, the average Dream for All-contingent mortgage pre-approval recorded at application, the average mortgage taken out when buying unassisted, and the difference between the two (pre-approval minus unassisted mortgage). Each bar is a two-stage least squares estimate from the regression system in equation 2; whiskers are 95% confidence intervals. Marginal buyers rent rather than buy unassisted, so their unassisted mortgage is zero and the difference equals their pre-approval.

Figure 10: Effects of Dream for All utilization on local labor market and neighborhood quality



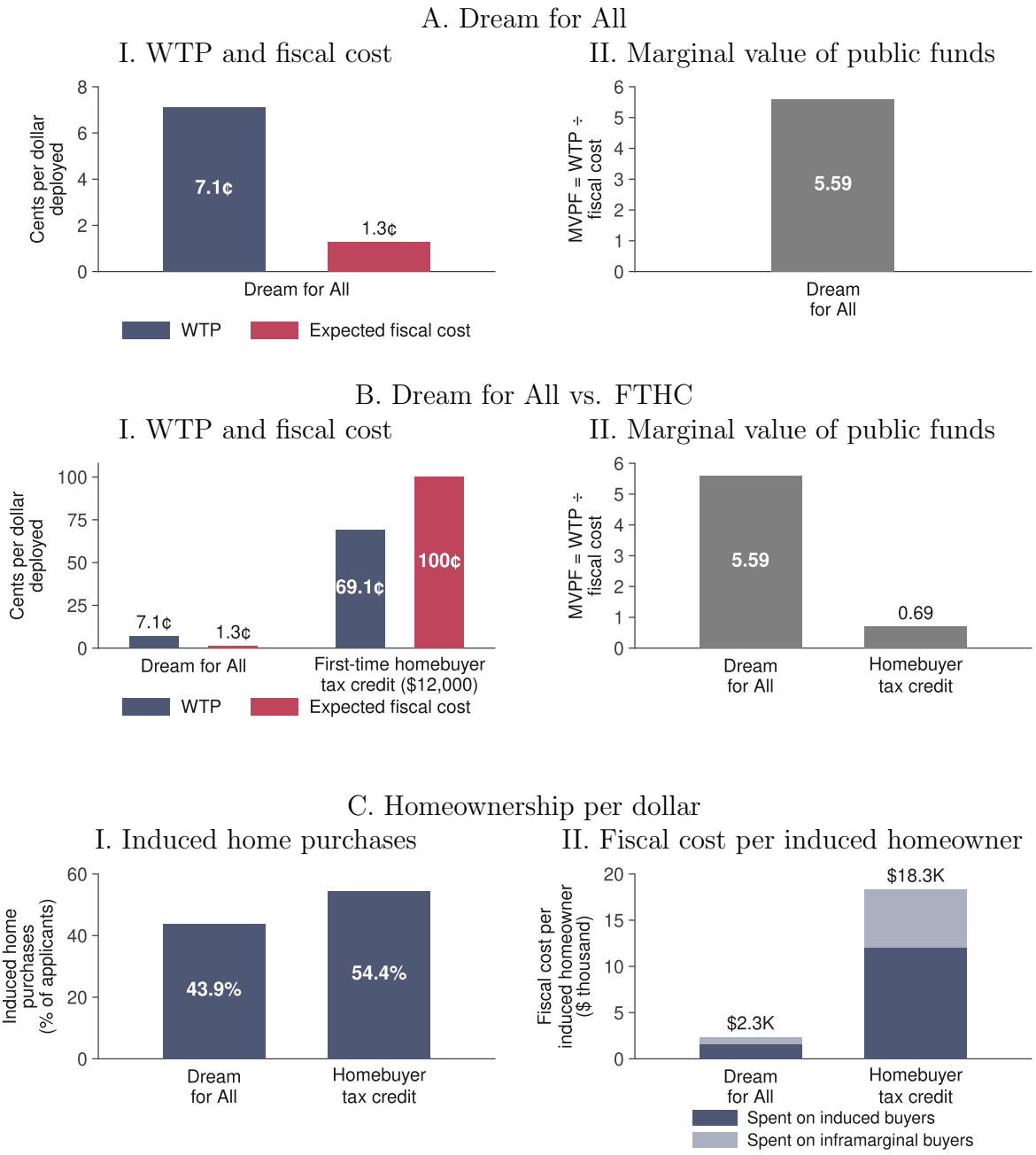
Notes: This figure shows effects of Dream for All utilization on neighborhood quality for inframarginal takers and marginal buyers. Each outcome is a characteristic of the Census tract in which the applicant lives six quarters after the lottery, standardized by the control group's pre-lottery mean and standard deviation and oriented so that higher values indicate a higher-amenity neighborhood. Red diamonds are effects of buying with Dream for All relative to buying unassisted for inframarginal takers, $(\mu_3 \div \pi_3) - (\mu_2 \div \pi_2)$ from systems 4 and 5; blue circles are effects of buying with Dream for All relative to renting for marginal buyers, $(\mu_3 \div \pi_3) - (\mu_1 \div \pi_1)$ from systems 3 and 5. Horizontal bars are 95% confidence intervals from the nonparametric block bootstrap.

Figure 11: Model fit at baseline parameters



Notes: This figure shows how well the estimated model of Section 7 matches the empirical moments targeted in estimation. Red diamonds are moments generated by the model at the estimated parameters in Table 2; navy circles and bars are the corresponding empirical moments and their 95% confidence intervals. Panel A shows applicant type shares (the renter never-taker share is one minus the three shown). Panels B and C show the distribution of cash down payments as a share of purchase price among inframarginal takers buying unassisted and among buyer never-takers; within each panel the four bins sum to one. The estimation targets the nine independent moments among these eleven. The overidentification test in the upper right treats the binding wealth restriction as an equality constraint and has 3 degrees of freedom; it rejects the model at the 1% level.

Figure 12: Willingness to pay, fiscal cost, MVPF, and homeownership gains



Notes: This figure shows the paper's main welfare results: applicants' willingness to pay for Dream for All, the program's expected fiscal cost, and their ratio, the MVPF, alongside the same quantities for a counterfactual transfer offered to the same applicants. All quantities are computed in the model at the parameters of Table 2. Panel A.I shows average willingness to pay (equivalent variation) for Dream for All and the state's expected fiscal cost, each in cents per dollar of Dream for All principal deployed; Panel A.II shows their ratio, the MVPF. Panel B repeats Panel A alongside a counterfactual First-Time Homebuyer Credit, a $12,000 grant at closing offered to the same applicants, with the vertical axis of B.I rescaled. Panel C.I shows the share of applicants each program induces to buy a home who would otherwise rent; Panel C.II shows each program's expected fiscal cost per induced homeowner, in thousands of dollars, split between dollars spent on applicants induced to buy and dollars spent on applicants who would have bought without assistance.

Table 1: Dream for All utilization effects: purchase terms and property characteristics

	Counterfactual:	Dream for All effect	
	Buying unassisted	Estimate	95% CI
	(1)	(2)	(3)
Panel A. Financing			
Mortgage size	555,541	−74,210	(−102,686, −45,734)
Mortgage payment	4,116	−342	(−559, −125)
% First mortgage-to-value	89.7	−9.3	(−10.0, −8.6)
% Combined loan-to-value	89.7	9.9	(9.2, 10.6)
Panel B. Housing consumption			
Price paid	639,882	−18,181	(−38,563, 2,201)
Building size (square feet)	1,450.8	−79.9	(−136.2, −23.7)
Number rooms	6.1	−0.3	(−0.5, 0.0)
Number bedrooms	2.9	−0.1	(−0.2, −0.1)
Number bathrooms	2.0	0.0	(−0.1, 0.1)

Notes: This table shows the effects of Dream for All utilization on inframarginal takers' home purchase financing (Panel A) and housing consumption (Panel B), relative to the terms on which they would have bought unassisted. Column (1) reports the average outcome of inframarginal takers when buying unassisted, the IV coefficient $\mu_2 \div \pi_2$ from system 4. Column (2) reports the effect of buying with Dream for All relative to buying unassisted for inframarginal takers, $(\mu_3 \div \pi_3) - (\mu_2 \div \pi_2)$, under the representation-in-observables assumption; column (3) reports 95% confidence intervals from the nonparametric block bootstrap. Mortgage size and monthly payment (principal, interest, taxes, and insurance) are from credit records; price paid and property characteristics are from property records. First mortgage-to-value and combined loan-to-value ratios are relative to the purchase price; the combined ratio includes the Dream for All loan.

Table 2: Model parametrization

Parameter	Description	Value/Parametrization
	A. Preferences	
$\bar{h}$	Housing requirement (units of annual income)	4.5
β	Weight on housing above the requirement	0.01
	B. House-price process	
π	Probability the house gains value	0.761
p_H	Price in the gain state (annualized, $p_0 = 1$)	1.048
p_L	Price in the loss state (annualized, $p_0 = 1$)	0.888
ρ	State's discount rate	0.05
	C. Type distribution (estimated)	
F	Joint distribution of log wealth and log preference for homeownership	Bivariate normal, $\mu_w = 0.092$, $\sigma_w^2 = 4.10$, $\mu_\phi = -2.125$, $\sigma_\phi^2 = 9.49$, $\varrho = -0.960$
$G_{w,\phi}$	Probability the voucher goes unused given w, ϕ ($\kappa \in \{0, \infty\}$)	$\Pr(\kappa = \infty \mid w, \phi) = \Phi\big(a + b\,[\log \phi - \log \bar{\phi}(w)]\big)$, $a = -1.896$, $b = 2.16$

Notes: This table shows the parameter values used in the quantitative model: calibrated preference and financial environment parameters (Panel A), the calibrated home price process (Panel B), and the estimated joint distribution of wealth, preference for homeownership, and hassle cost (Panel C). Income and the period-0 house price are normalized to 1, so wealth, housing, and prices are in units of annual income. Panel A parameters are calibrated as described in the text. Panel B parameters are calibrated to purchase-price-weighted annualized repeat-sale returns in California, 1988–2023. Panel C parameters are estimated by classical minimum distance, targeting the four applicant type shares and the down payment share distributions of inframarginal takers buying unassisted and of buyer never-takers, subject to the restriction that the median wealth of marginal buyers equals that of the least wealthy third of inframarginal takers.$\bar{\phi}(w)$ is the preference for homeownership at which an agent with wealth w is indifferent between renting and buying unassisted.

Homeownership and Government-Provided Liquidity

Online Appendix

Appendix Contents

A Supplemental exhibits **67**

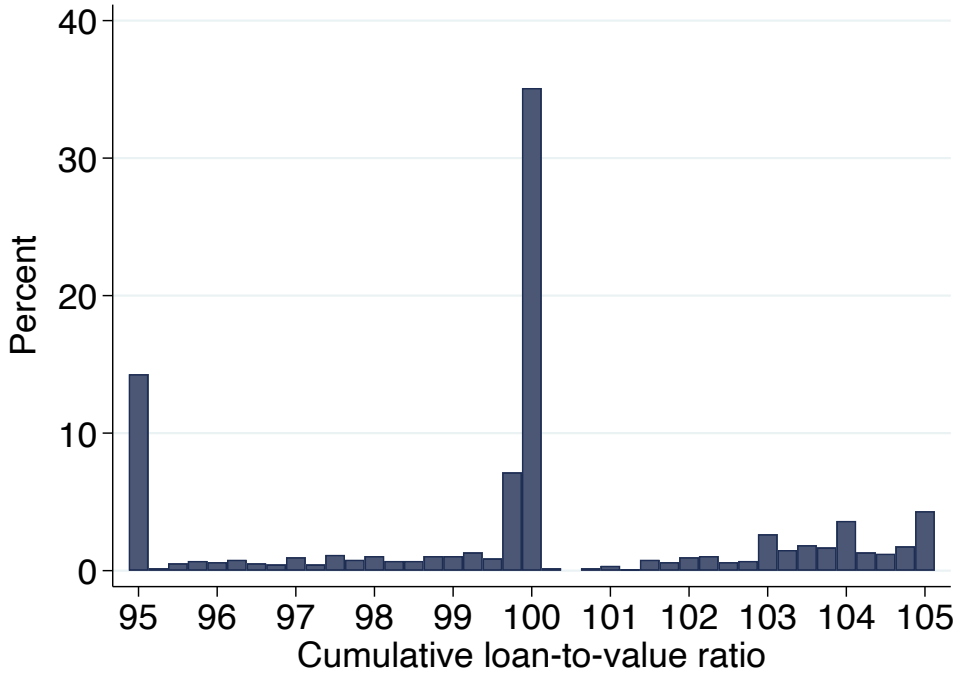
B Merging CalHFA and UCCCP data **83**

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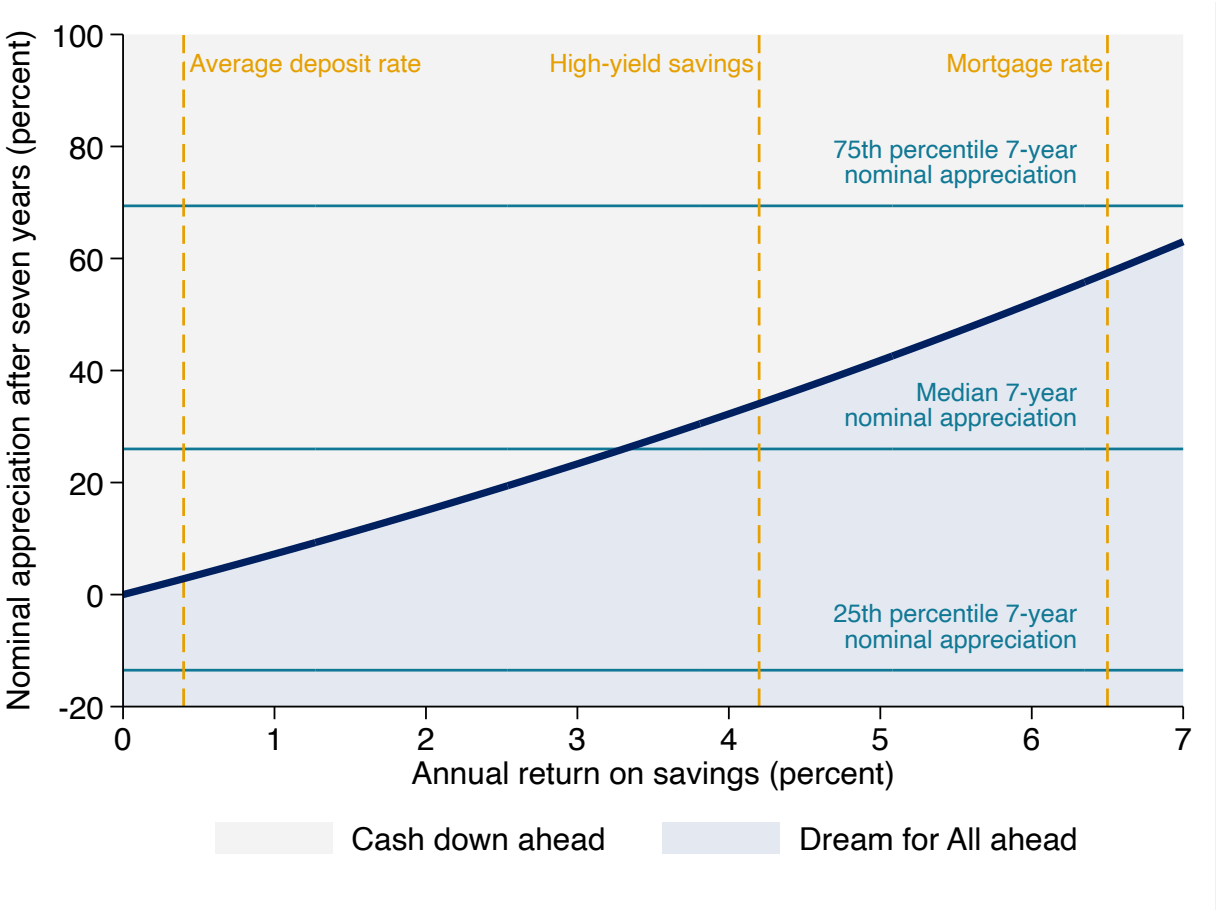
A Supplemental exhibits

Figure A.1: Combined loan-to-value ratio distribution for Dream for All-assisted home purchases



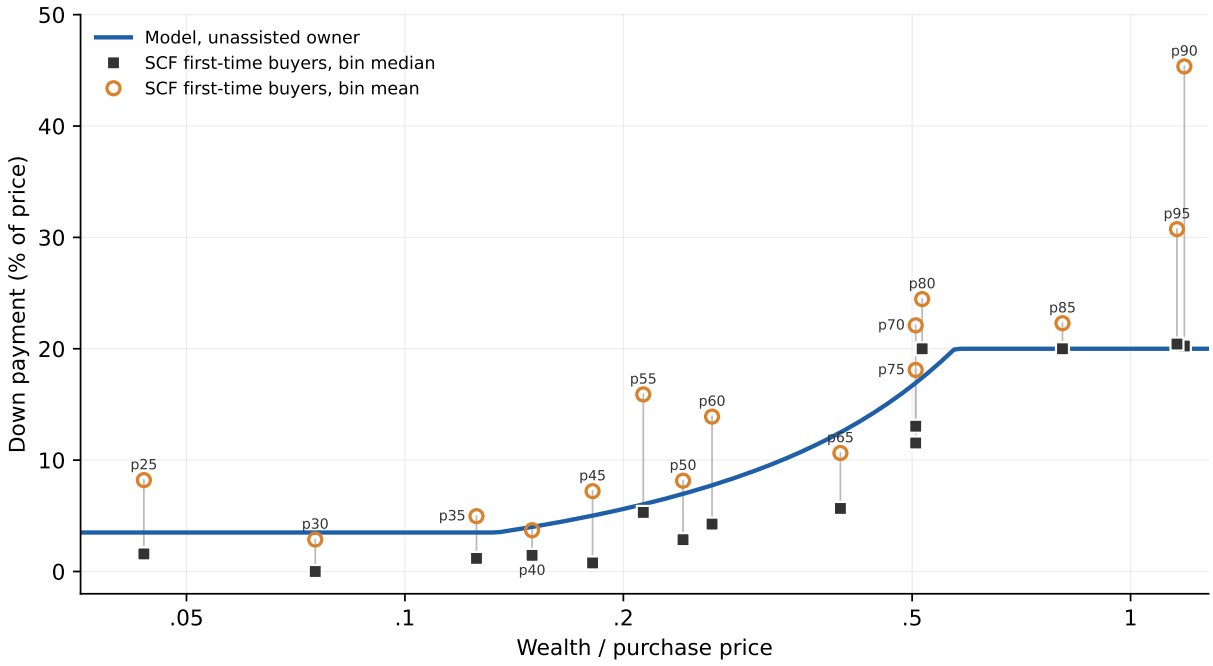
Notes: This figure shows the distribution of combined loan-to-value ratios (first mortgage plus Dream for All subordinate loan, divided by purchase price) among home purchases financed with Dream for All loans. The combined loan-to-value ratio is constrained by the program to fall between 95% and 105%. The modal ratio is 100%, consistent with borrowers making zero cash down payments and using Dream for All financing to cover closing costs up to 5% of the purchase price. Data are from California Housing Finance Agency loan utilization records.

Figure A.2: Buying a $500K home with Dream for All vs. 20% cash down payment



Notes: The line shows the seven-year nominal appreciation at which a buyer with $100K in cash is indifferent between financing a 20% down payment on a $500K home with Dream for All and paying it in cash, as a function of the annual return on her savings. Both buyers take an 80% first mortgage, so the comparison is between the interest the Dream for All buyer earns on the $100K she keeps (compounded monthly for seven years) and the 20% of appreciation she owes at sale; below the line Dream for All leaves her with more at sale, above it cash does. Horizontal lines mark the 25th percentile, median (26%), and 75th percentile of seven-year nominal appreciation among California homes purchased in 1988 or later in the ATTOM recorder data, weighted by purchase price. Vertical lines mark the FDIC national average savings deposit rate (0.4%, September 2026), the top high-yield savings rate (4.2%, October 2026), and a 6.5% mortgage rate.

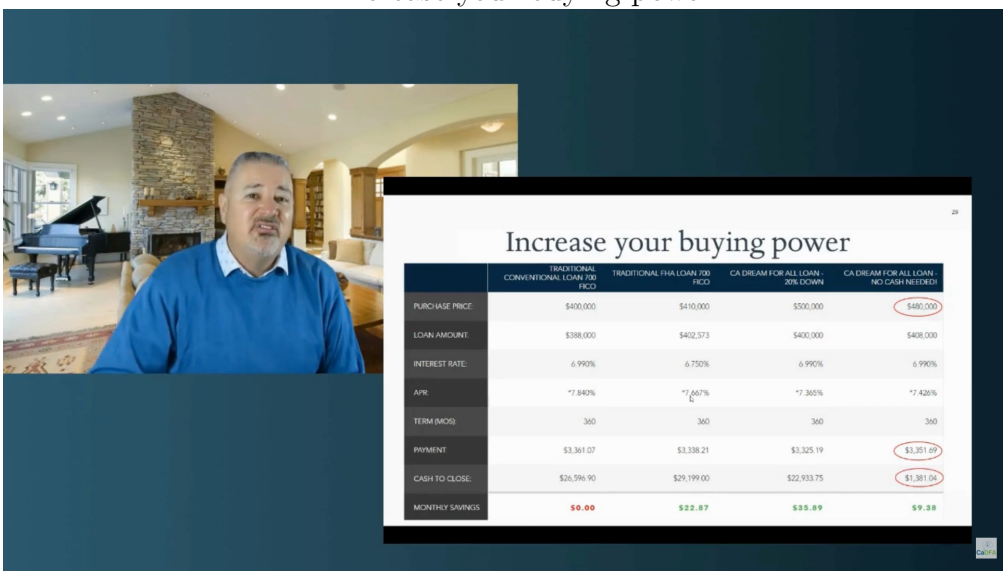
Figure A.3: Down payment shares of first-time homebuyers, by non-housing wealth: model and data



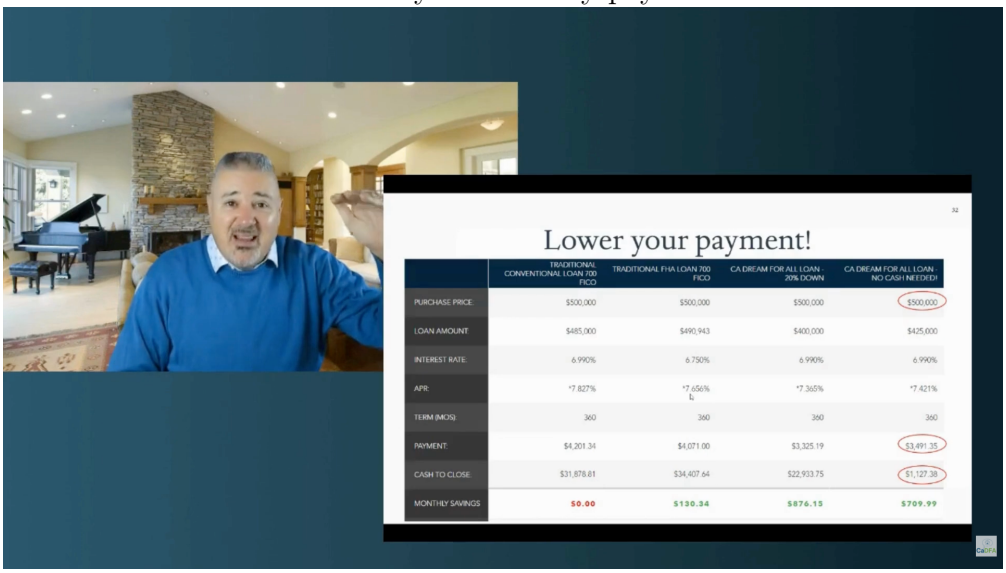
Notes: Markers show the median (squares) and mean (circles) cash down payment as a share of the purchase price among first-time homebuyers in the 1995–2022 waves of the Survey of Consumer Finances who bought in the survey year or the year before, grouped into twenty bins of pre-purchase non-housing wealth (net worth excluding the home, with the down payment added back). Each bin is plotted at the ratio of its median wealth to its median purchase price; labels give the bin's lower percentile (p70 is the 70th–75th percentile). Bins whose median wealth is below 3.5% of their median purchase price, the minimum down payment in the model, are not shown. The line is the down payment share chosen by an unassisted buyer in the model at each wealth, with wealth divided by that buyer's purchase price, at the parameters of Table 2.

Figure A.4: Advertised benefits of Dream for All

A. Increase your buying power

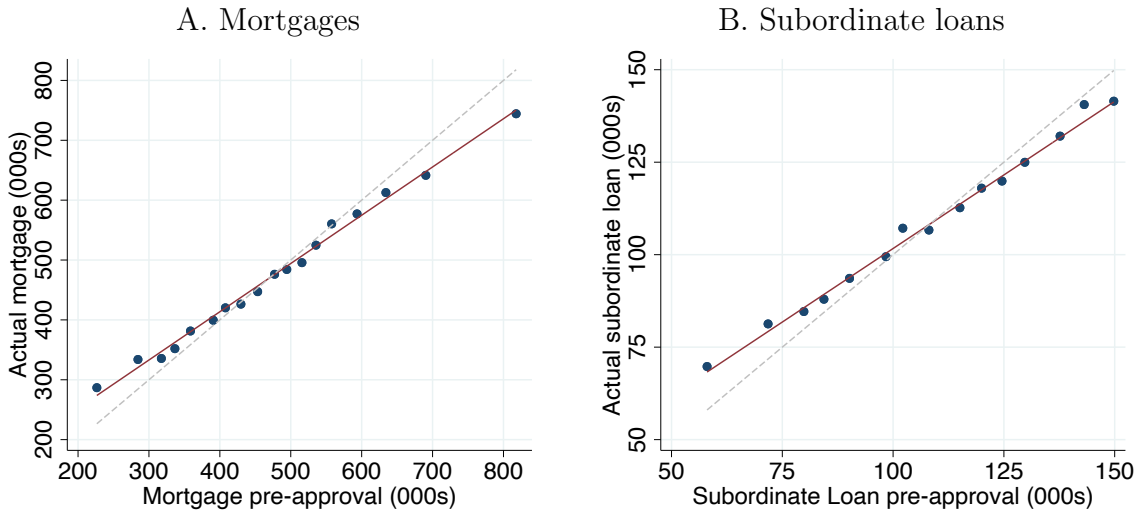


B. Lower your monthly payment



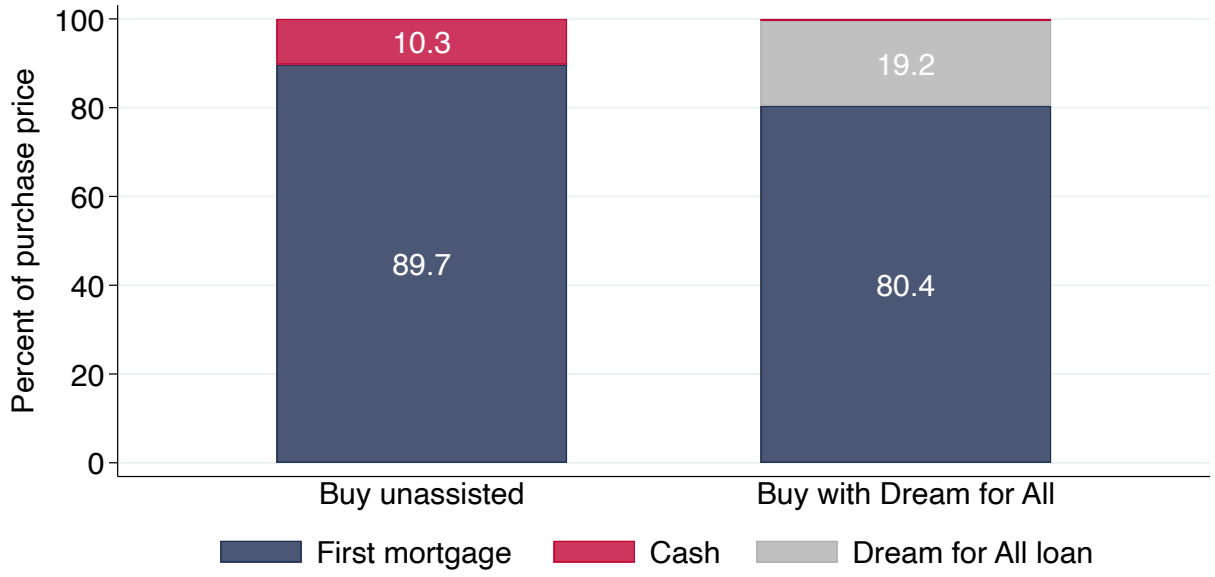
Notes: Screenshots accessed from https://www.youtube.com/watch?v=t7pcKu_0MDo&list=PLeLgXh0X1mPnKrMP3CgKb37Q6uexO19sk&index=4.

Figure A.5: Loan pre-approval vs. loan utilization among Dream for All-assisted home purchases



Notes: Panel A plots actual first mortgage amounts against first mortgage pre-approval amounts for Dream for All-assisted home purchases. Panel B plots actual subordinate (Dream for All) loan amounts against subordinate loan pre-approval amounts. The solid red line is a linear fit; the dashed gray line is the 45-degree line. Pre-approval amounts are from Dream for All applications; actual loan amounts are from CalHFA loan utilization records. The strong correlation, particularly for subordinate loans, supports using pre-approval amounts as proxies for counterfactual loan utilization in the complier characteristics analysis of Figure 9.

Figure A.6: Effects of Dream for All utilization on financing shares

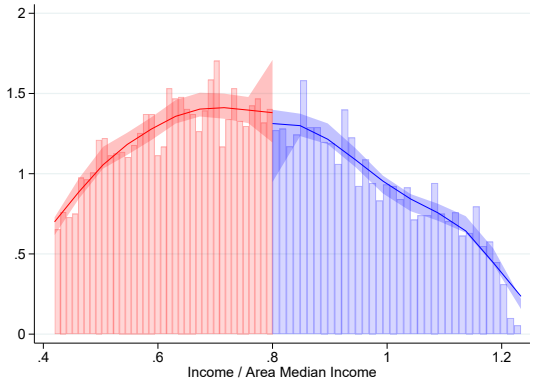


Notes: Bars decompose inframarginal takers' average home purchase price into first mortgage, cash, and Dream for All loan when buying unassisted (left) and with Dream for All (right), using the estimates in Table 1. The first-mortgage share is the first mortgage loan-to-value ratio, the Dream for All share is the difference between the combined and first-mortgage loan-to-value ratios, and cash is the remainder; the left bar uses column (1) and the right bar adds the effects in column (2). Cash when buying with Dream for All is 0.4% of the purchase price and is not labeled.

Figure A.7: Effect of Dream for All cost jump on take-up and housing consumption

A. Manipulation tests: income-to-AMI ratio

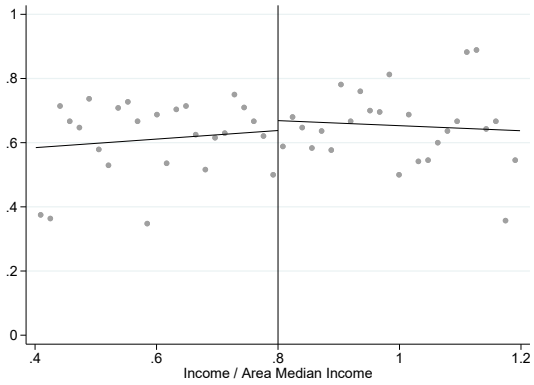
I. Experimental sample



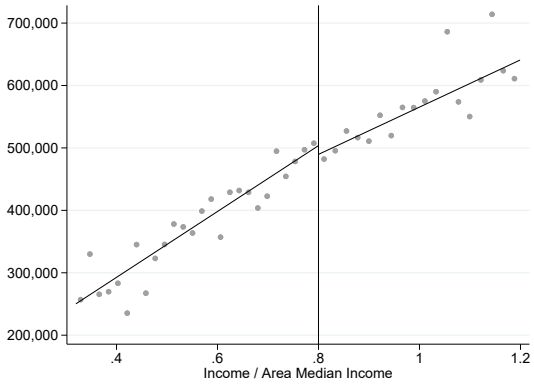
II. Dream for All user sub-sample



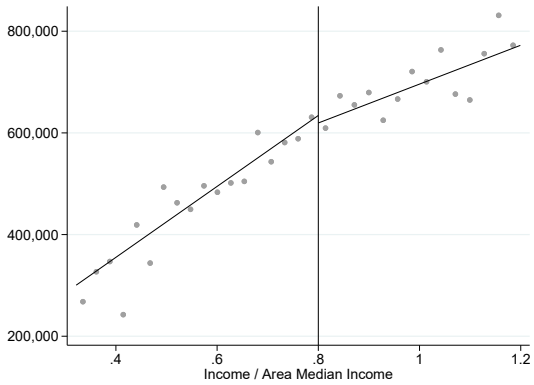
B. Effect of cost jump on take-up (Experimental sample)



C. Effect of cost jump on mortgage size (Dream for All user sub-sample)

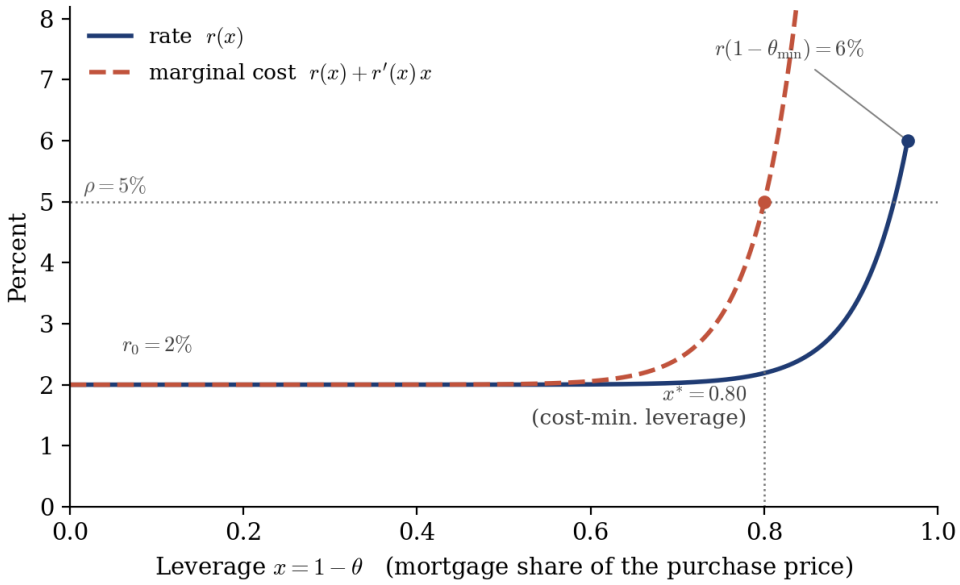


D. Effect of cost jump on home purchase price (Dream for All user sub-sample)



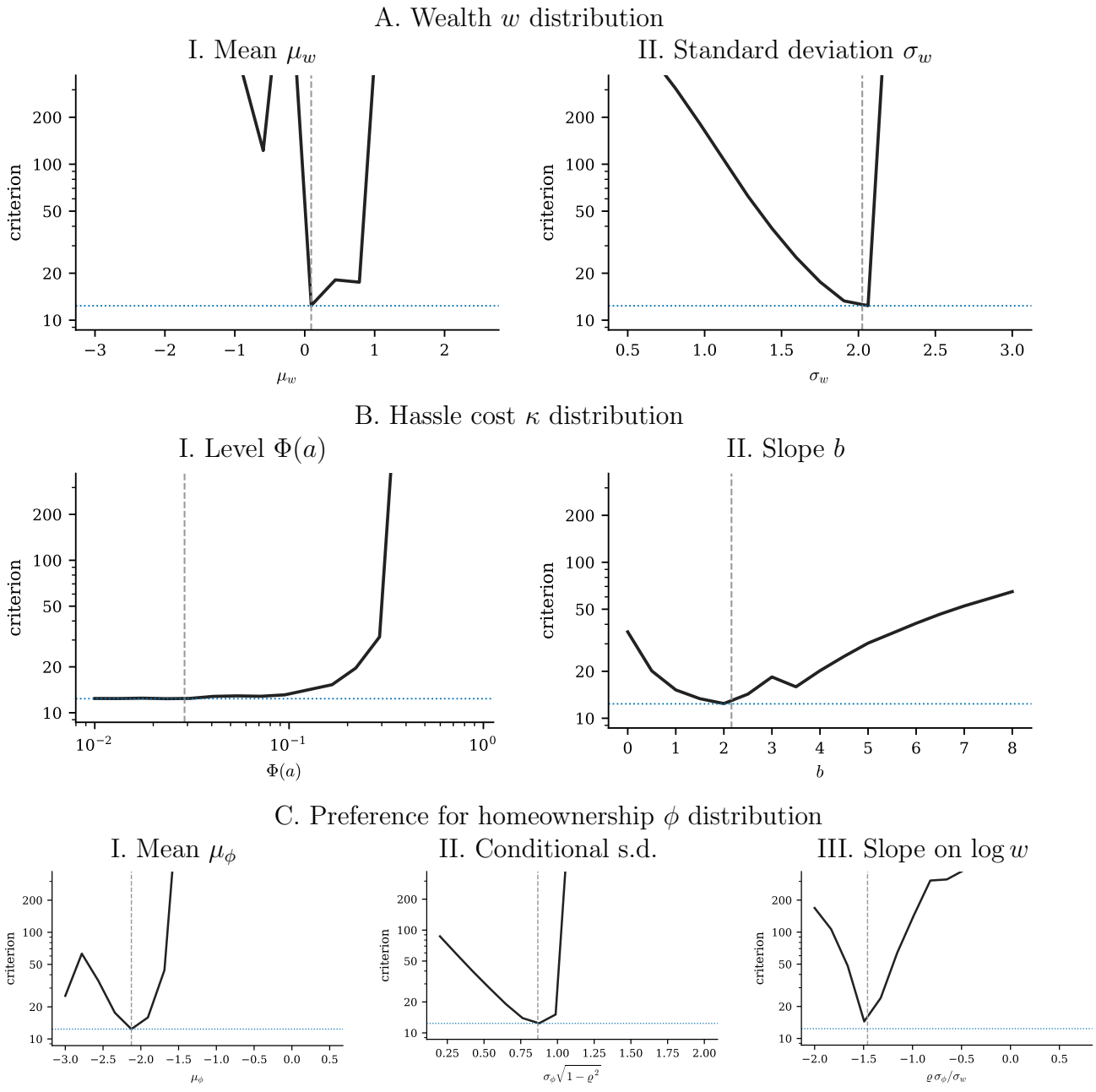
Notes: This figure tests for responses to the discontinuous jump in shared appreciation loan cost at the 80% AMI threshold. Panel A shows manipulation tests of Cattaneo, Jansson and Ma (2018) for the running variable around the 80% cutoff. Panels B–D show binned scatterplots constructed via the IMSE-optimal procedure of Calonico, Cattaneo and Titiunik (2015), overlaid with local linear fits, of effects of the cost jump on Dream for All take-up, mortgage size, and home purchase price. Plots are implemented using Calonico, Cattaneo, Farrell and Titiunik (2017) `rdplot` with MSE-optimal bandwidth and triangular kernel.

Figure A.8: Calibrated mortgage rate schedule



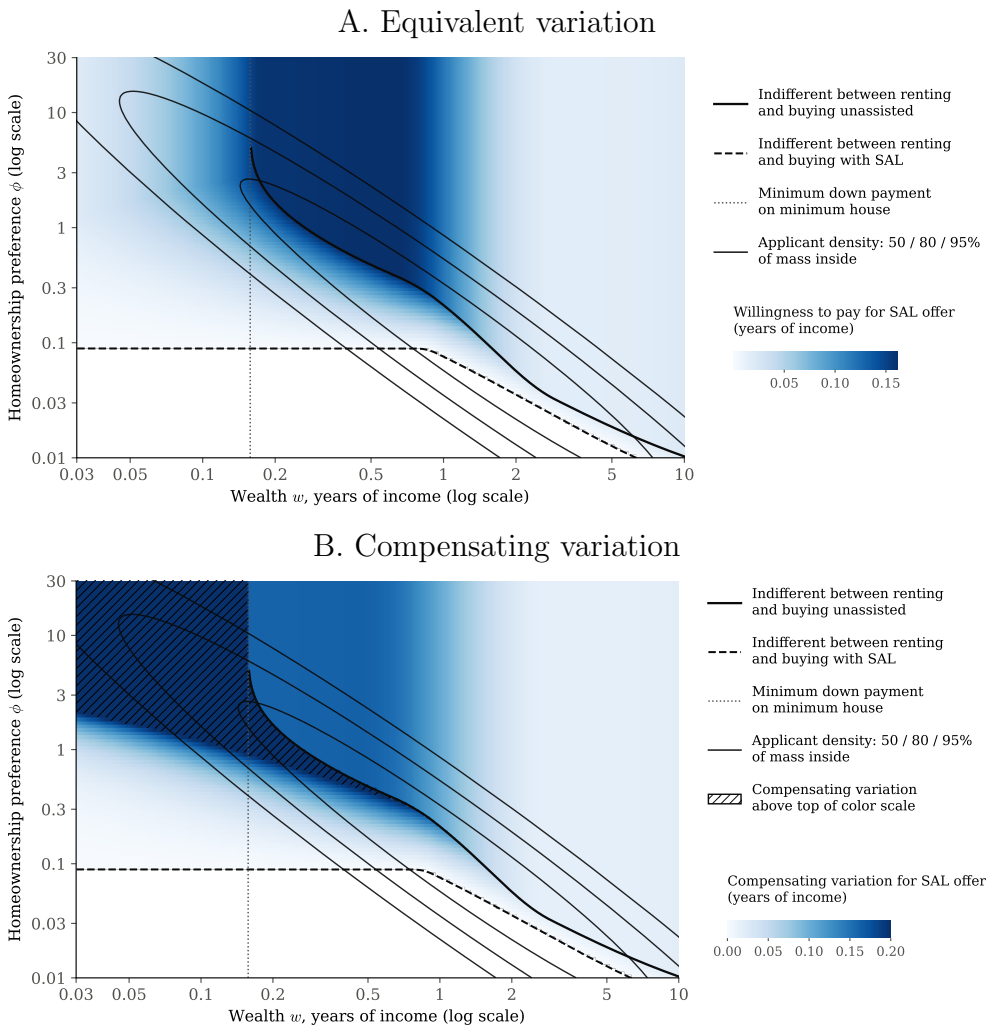
Notes: The solid line is the mortgage interest rate as a function of the mortgage share of the purchase price, $x = 1-\theta$, and the dashed line is the marginal cost of mortgage financing, $r(x)+r'(x)x$, the left-hand side of equation (1). The schedule is exponential in the mortgage share, $r(x) = r_0 + a(e^{kx}-1)$, with $r_0 = 2\%$ and a and k set by two conditions: the marginal cost equals the household return on savings $\rho = 5\%$ at $x = 0.8$, so that a 20% down payment is cost-minimizing, and the rate at the maximum mortgage share $1-\underline{\theta} = 0.965$ is 6%, one percentage point above ρ.

Figure A.9: Identification of type distribution parameters



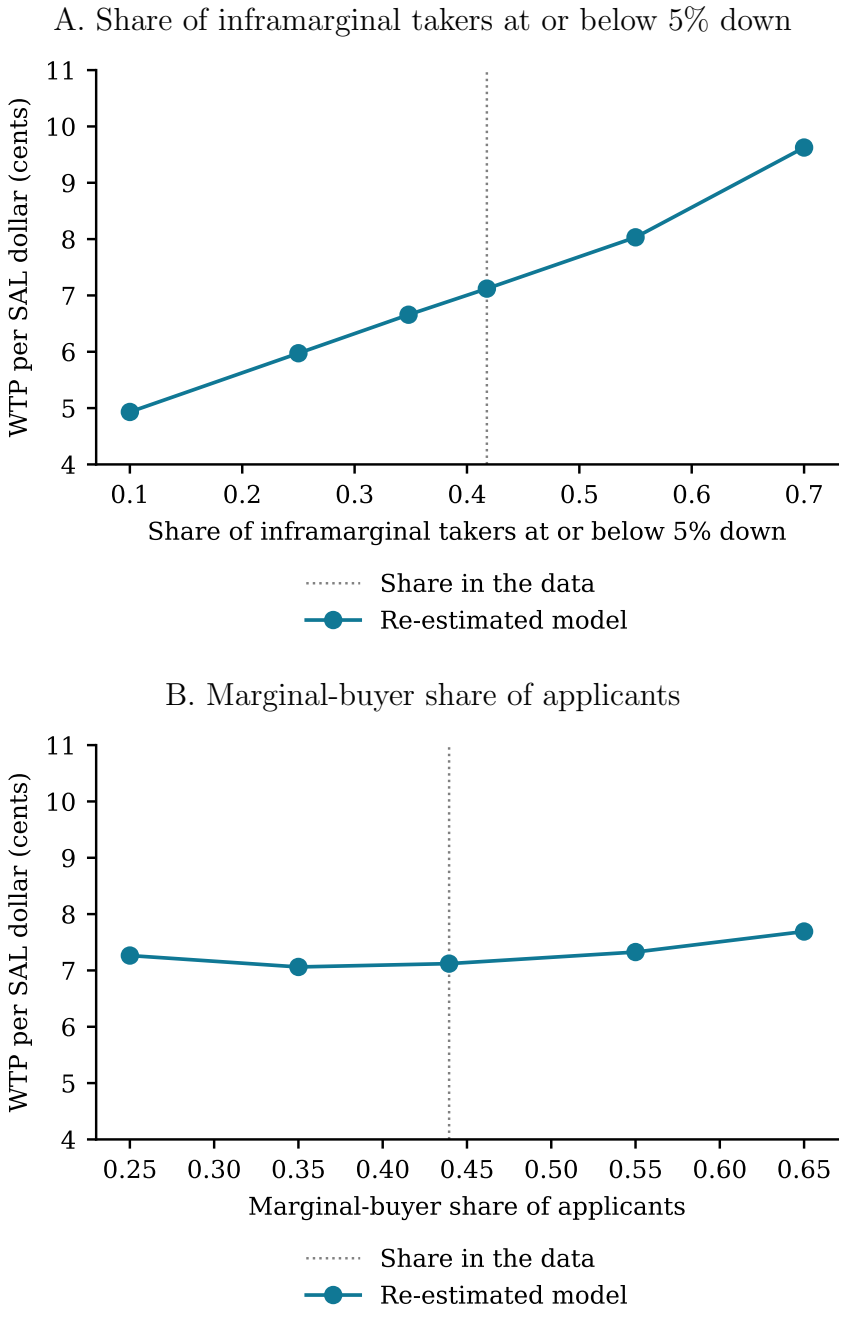
Notes: Each panel plots the minimum-distance criterion as one parameter varies, re-optimizing the other six at each value subject to the wealth restriction; the dashed line marks the estimate in Table 2 and the dotted line the criterion minimum. Panel C varies the conditional distribution of $\log \phi$ given $\log w$: its mean, its conditional standard deviation $\sigma_\phi\sqrt{1-\varrho^2}$, and its slope $\varrho\sigma_\phi/\sigma_w$, which given σ_w map one-to-one into $(\mu_\phi, \sigma_\phi^2, \varrho)$.

Figure A.10: Willingness to pay for Dream for All by applicant type



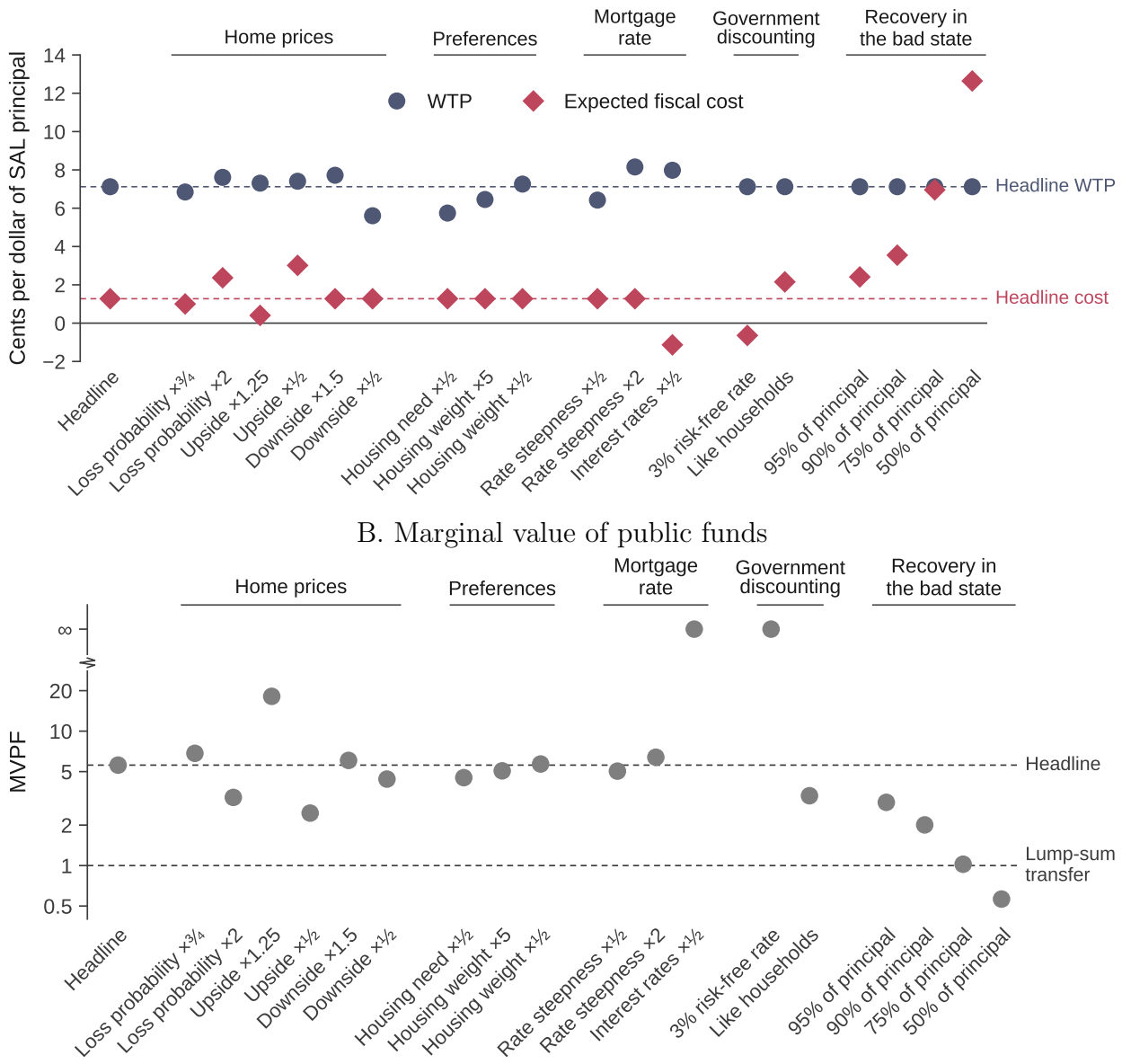
Notes: Both panels show the type space of Figure 4 Panel A at the parameters of Table 2, with wealth and preference for homeownership on log scales and wealth in years of income. Color is willingness to pay for a Dream for All offer in years of income: in Panel A the equivalent variation, the wealth an applicant who buys with Dream for All would give up before preferring her fallback (renting for marginal buyers, buying unassisted for inframarginal takers), which cannot exceed her wealth; in Panel B the compensating variation, the wealth she would need in the fallback to be as well off as with Dream for All. Applicants below the dashed line do not take the loan and are left blank. Thin solid curves enclose 50, 80, and 95 percent of the estimated applicant density. In Panel B the color scale runs from 0 to 0.2 years of income; hatched cells exceed it (compensating variation is capped at 60 years of income) and are marginal buyers with a strong preference for ownership, whose fallback is renting.

Figure A.11: WTP for Dream for All when the model is re-estimated on counterfactual moments



Notes: Each point re-estimates the model with one target replaced and everything else (calibration, standard errors, wealth restriction) unchanged. In Panel A the share of inframarginal takers who would put 5% or less down without the SAL is set to the value on the horizontal axis, the remaining three bins are rescaled proportionally, and the corresponding bin for buyers who declined the SAL is scaled by the same factor. In Panel B the marginal-buyer share of applicants is set to the value on the horizontal axis, with renter never-takers absorbing the change. The dotted line marks the value in the data. At the leftmost point of Panel A the model cannot match the target (it reaches .16); at the rightmost the estimated non-use probability no longer varies with ownership preference.

Figure A.12: Sensitivity of WTP, fiscal cost, and MVPF to model assumptions



Notes: Each point is one alternative to the baseline, at 1:1 appreciation sharing. Rows under home prices, preferences, and mortgage rate re-estimate the model under the alternative assumption; rows under government discounting and recovery hold the baseline estimates fixed and change only the state's cost. Home prices: good-state price appreciation multiplied by 1.25 or 0.5 (6.0% or 2.4% versus 4.8%); bad-state loss multiplied by 1.5 or 0.5 (16.9% or 5.6% versus 11.2%); probability of the bad state multiplied by 0.75 or 2 (17.9% or 47.7% versus 23.9%). Preferences: minimum housing need halved; utility weight on housing multiplied by 5 or 0.5. Mortgage rate: the rate schedule's slope halved or doubled holding its midpoint; all interest rates (household discount rate, state discount rate, mortgage rates) halved. Government discounting: state discount rate 3% rather than 5%; state discounts payoffs with takers' marginal utilities. Recovery: state recovers the stated share of principal in the bad state. Where expected fiscal cost is negative the MVPF is undefined and plotted at ∞.

Table A.1: Net financial position at sale on $500K purchase for an unconstrained buyer

Appreciation (nominal)	80% mortgage, 20% Dream for All	80% mortgage, 20% cash	96.5% mortgage, 3.5% cash
50%	$459K	$468K	$423K
−10%	$209K	$168K	$123K

Notes: Each cell is the buyer's net financial position at sale seven years after purchasing a $500K home: home value less the remaining mortgage balance and any Dream for All repayment, plus her savings balance. The buyer has $100K in cash; whatever she does not put down she keeps in savings earning 5%, compounded monthly. All three mortgages are 30-year fully-amortizing loans at 6.5% interest; the 96.5% mortgage is an FHA loan with a 1.75% upfront mortgage insurance premium, financed, and an annual premium of 0.55% of the balance. To keep the columns comparable, every buyer spends the same amount per month, the FHA buyer's payment, and the two 80% mortgage buyers save the difference. The Dream for All buyer repays the $100K principal plus 20% of any appreciation at sale.

Table A.2: Analysis sample baseline characteristics

	Pooled	Control		Treatment		p-value of
	Mean	Mean	SD	Mean	SD	T - C difference
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Reported in Dream for All Pre-Registrations						
Age	40.5	40.5	10.7	40.6	10.9	0.75
% Female	40.7	40.7	49.1	40.8	49.2	0.49
Household size	2.9	2.9	1.4	2.9	1.4	0.62
% Race: American Indian or Alaska Native	1.2	1.1	10.6	1.3	11.1	0.72
% Race: Asian or Pacific Islander	34.3	34.8	47.6	31.4	46.4	0.99
% Race: Black or African American	9.2	9.3	29.0	9.1	28.8	0.97
% Race: White	28.8	28.4	45.1	31.7	46.6	0.50
% Race: Other	19.7	19.8	39.8	18.9	39.2	0.21
% Race: Decline to answer	6.8	6.7	25.0	7.5	26.4	0.44
% Ethnicity: Hispanic or Latino	28.8	28.0	44.9	34.1	47.4	0.51
Total credit qualifying income	121,286	122,135	68,308	115,654	57,852	0.11
First mortgage loan pre-approval	511,869	514,116	172,376	496,989	180,959	0.45
Subordinate loan pre-approval	121,034	121,400	29,771	118,615	30,877	0.24
Panel B. Recorded in University of California Consumer Credit Panel as of 2024 Q1						
Credit score	753.7	754.0	51.9	751.5	51.0	0.46
Total credit limit	26,416	26,358	25,104	26,797	26,548	0.10
Number of open auto loans	0.5	0.5	0.6	0.5	0.7	0.48
Number of open credit cards	4.3	4.3	3.4	4.4	3.5	0.50
Revolving credit utilized	11,128.6	11,037.6	19,054.8	11,730.6	19,431.9	0.32
Number of open student loans	1.1	1.1	3.0	1.2	3.2	0.14
% Any current delinquencies	0.6	0.6	7.9	0.5	7.1	0.45
% Any delinquency since 2018	25.3	25.1	43.4	26.4	44.1	0.96
% Any delinquency since 2020	18.8	18.7	39.0	19.6	39.7	0.86
% Any delinquency since 2022	11.8	11.8	32.2	11.7	32.2	0.40
% Any public record	2.0	2.0	14.1	2.1	14.3	0.68
Number of observations	9,095	7,901		1,194		
F-test		F-statistic	p-value			
		0.82	0.70			

Notes: This table shows baseline characteristics of the analysis sample. Column (1) reports pooled means across treatment and control groups. Columns (2)–(3) report means and standard deviations for the control group (applicants not selected in the lottery), and columns (4)–(5) report the same for the treatment group (applicants initially offered a Dream for All loan). Column (6) reports p-values from regressions of each variable on a treatment indicator controlling for region fixed effects, with heteroskedasticity-robust standard errors. Panel A variables are from Dream for All pre-registration applications. Race categories are not mutually exclusive. Total credit qualifying income sums income across all co-borrowers on an application. First mortgage and subordinate loan pre-approval amounts are from CalHFA-approved lenders. Panel B variables are from the University of California Consumer Credit Panel (UCCCP) as of 2024Q1. The F-test in the bottom row tests the joint null that all coefficients on the treatment indicator are zero.

Table A.3: Complier characteristics

	Sample Mean (1)	Compliers Mean (2)	Inframarginal Mean (3)	Marginal Mean (4)	p-value of (3) - (4) difference (5)
Panel A. Reported in Dream for All pre-registrations					
Age	40.5	40.4	39.1	40.7	0.04
% Female	40.7	38.5	34.7	42.7	0.02
Household size	2.9	3.0	3.2	3.0	0.07
% Race: Asian or Pacific Islander	34.3	32.9	32.2	33.8	0.63
% Race: Black or African American	9.2	8.7	6.9	9.5	0.19
% Race: White	28.8	31.3	32.2	28.2	0.22
% Ethnicity: Hispanic or Latino	28.8	34.7	35.0	31.4	0.27
Total credit qualifying income	121,286	114,469	126,317	117,076	0.03
First mortgage loan pre-approval	511,869	502,731	519,467	492,026	0.03
Subordinate loan pre-approval	121,034	120,165	121,692	118,217	0.11
Panel B. Recorded in University of California Consumer Credit Panel as of 2024 Q1					
Credit score	753.7	756.2	758.4	752.3	0.13
Total credit limit	26,416	26,515.7	24,432.4	24,703.5	0.88
Number of open auto loans	0.5	0.5	0.5	0.5	0.14
Number of open credit cards	4.3	4.4	4.4	4.2	0.60
Number of open student loans	1.1	1.2	1.1	0.9	0.50
Revolving credit utilization	11,128.6	10,853.1	7,877.4	10,973.1	0.03
% Any delinquency since 2018	25.3	23.7	27.8	21.8	0.06
% Any delinquency since 2020	18.8	17.2	21.4	15.8	0.05
% Any delinquency since 2022	11.8	10.5	14.3	10.8	0.13
% Any public record	2.0	2.0	1.5	2.6	0.29
Share applicants	1.00	0.63	0.19	0.44	
Share compliers	–	1.00	0.30	0.70	

Notes: This table reports characteristics of the full analysis sample (column 1), all compliers (column 2), inframarginal compliers (c-compliers, column 3), and marginal compliers (n-compliers, column 4). Column (5) reports p-values from tests of equality between inframarginal and marginal complier means. Complier characteristics are estimated using the instrumental variables approach of Kline and Walters (2016). Inframarginal compliers are applicants who would buy a home without Dream for All assistance but use the program when offered; marginal compliers are applicants who would not buy a home without the program but purchase with Dream for All assistance when offered. The bottom rows report the share of all applicants in each group and the share of compliers in each subgroup. Complier shares are adjusted for region fixed effects. The n-complier share and c-complier share are estimated using equation 2. Outcomes are measured at baseline (2024Q1).

Table A.4: Implied effect of Dream for All utilization on mortgage default risk

	Monthly payment	CLTV
A. Effects of Dream for All on default risk factors		
Dream for All effect	−$342	+9.9 pp
Counterfactual: buying unassisted	$4,116	89.7%
Proportional effect	−8.3%	—
B. Estimated effects of risk factors on default		
Fuster and Willen (2017)	+1.6% per 1%	+1.85% per pp
Ganong and Noel (2020)	+1.2% per 1%	−0.55% per pp, assumed 0
C. Implied effects of Dream for All on default		
Fuster and Willen (2017)		
Hazard ratio, exp(A × B)	0.88	1.20
Net hazard ratio (product)	1.05	
Ganong and Noel (2020)		
Hazard ratio, 1 + A × B	0.90	1.00
Net hazard ratio (product)	0.90	

Notes: Panel A reproduces the effects of Dream for All utilization on inframarginal takers' monthly mortgage payment and combined loan-to-value ratio from Table 1; the payment is measured inclusive of taxes and insurance and the proportional reduction is applied as if to principal and interest, the payment concept in both papers. Panel B reports estimated effects of each risk factor on default from a proportional hazard model (Fuster and Willen, 2017) and a linear model (Ganong and Noel, 2020), whose statistically insignificant CLTV effect is set to zero. Panel C combines Panels A and B: under the proportional hazard model the hazard ratio for each factor is exp(A × B) and the net ratio is their product; under the linear model it is 1 + A × B.

B Merging CalHFA and UCCCP data

Approach. Our linkage strategy follows Fu, Davis, Rothstein, Ramesh and White (2022) and uses code adapted from their GitHub Repository. We merge the CalHFA and UCCCP data using PII fields common to the two data sets: Social Security Numbers, names, dates of birth, and addresses. To facilitate the merge, we hash these fields in the CalHFA data using the same hashing function used to mask these fields in the UCCCP. Before merging CalHFA and UCCCP data, we quarantine the original files received from CalHFA which have non-masked PII to preserve the de-identified nature of the UCCCP data. We also generate and hash partial fields in both data sets, including substrings and Soundex phonetic transcriptions, to facilitate fuzzy matching.[1]

We conduct five rounds of matching, each using a slightly different set of criteria to match observations across the two data sets. Table B.1 describes the criteria in each round. In general, the successive rounds grow less stringent in their match criteria, but the rounds are not structured so that each round's matches are a superset of the preceding round's.

To build our analysis data, we take the union of the matches from all rounds. When an individual in the CalHFA data matches to multiple individuals in the UCCCP data, we keep the UCCCP observation which matched in the earliest round and with the highest similarity score.

Results. Figure B.1 shows the merge success rate, disaggregating by matching round. 72.1% of individuals in the CalHFA data are matched to the UCCCP data using our linkage strategy. The most matches are attained in round 1, which enforces a perfect match on all PII fields except address, and round 3, which enforces a perfect match on date of birth and 2/3 of first name, last name, and address.

Figure B.2 shows the results of a regression of an indicator for whether we linked a record to the UCCCP on CalHFA applicants' individual characteristics. This figure suggests that match rates depend on individuals' demographic characteristics and the information available in the data. Individuals in the CalHFA data who stated their race as Black or American Indian or Alaska Native had slightly higher merge rates relative to Whites. Hispanic or Latino individuals had slightly lower merge rates, as did females and individuals in larger households. These patterns may have to do with the reliance of the linkage algorithm on names. Women in the US often change their names when they marry, making it more difficult to use names to link their records across data sets. The fact that individuals in larger households (i.e., non-single individuals) have lower merge rates is consistent with this hypothesis. In addition, Black and American Indian or Alaska Native individuals often have more distinctive names than Whites and Hispanics, making it easier to link their records across data sets.

Individuals listed as primary borrowers in the CalHFA program data set are somewhat

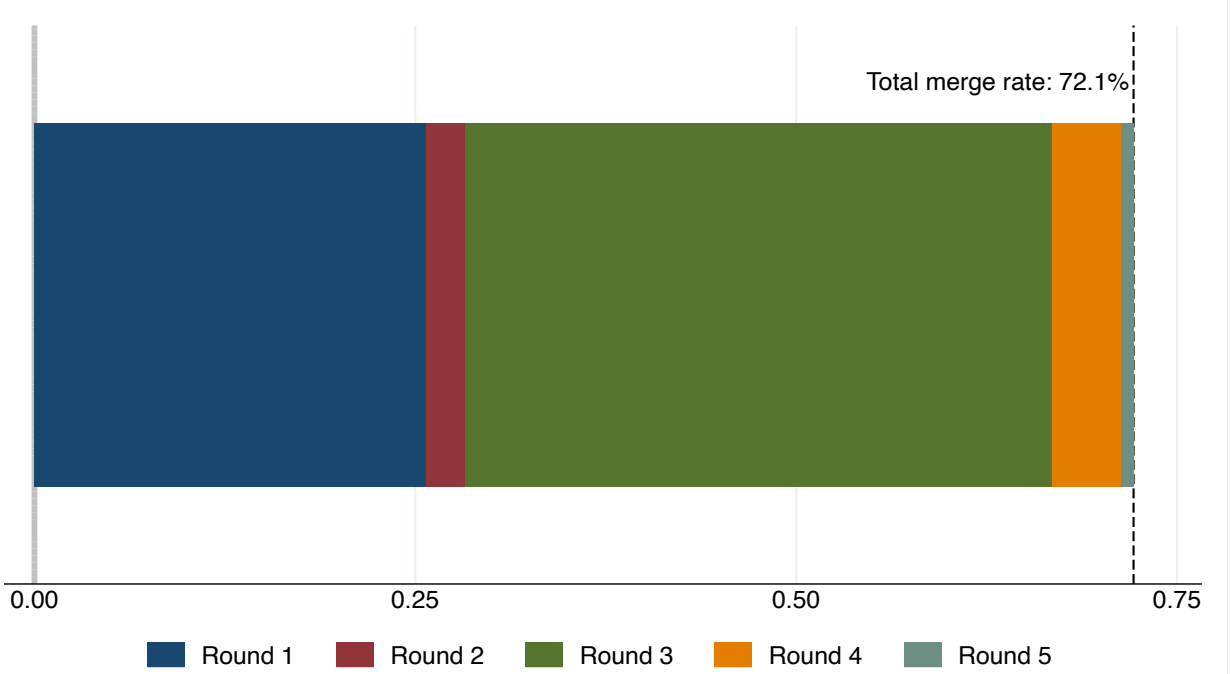
[1]Fuzzy matching with hashed linkage fields amounts to exact matching on partial fields using a scoring function which shows how to weight the importance of different partial field matches. For example, the fuzzy match scoring function we use gives a perfect match score if house numbers, street names, street suffixes, and unit IDs, city, state, and ZIP code all match across both datasets. It gives a slightly lower score if all of these fields match except for unit IDs; another lower score if all fields match except for unit IDs and street name suffixes; and so on until the lowest possible score in which only city and state match.

Table B.1: UCCCP-CalHFA matching rounds

Round	Description
1	Perfect match on first name, last name, and DOB, and strict fuzzy match on address.
2	Perfect match on first name, last name, and DOB, and loose fuzzy match on address.
3	Perfect match on DOB, and perfect match on 2/3 of first name, last name, and address. Drop if bad first name or last name.
4	Perfect match on DOB; perfect match on 1 of first name, last name, and address; and fuzzy match on the two fields that didn't have a perfect match. Drop if bad first name or last name.
5	Perfect match on first name and last name; strict fuzzy match on address and fuzzy match on DOB. Drop if bad DOB.

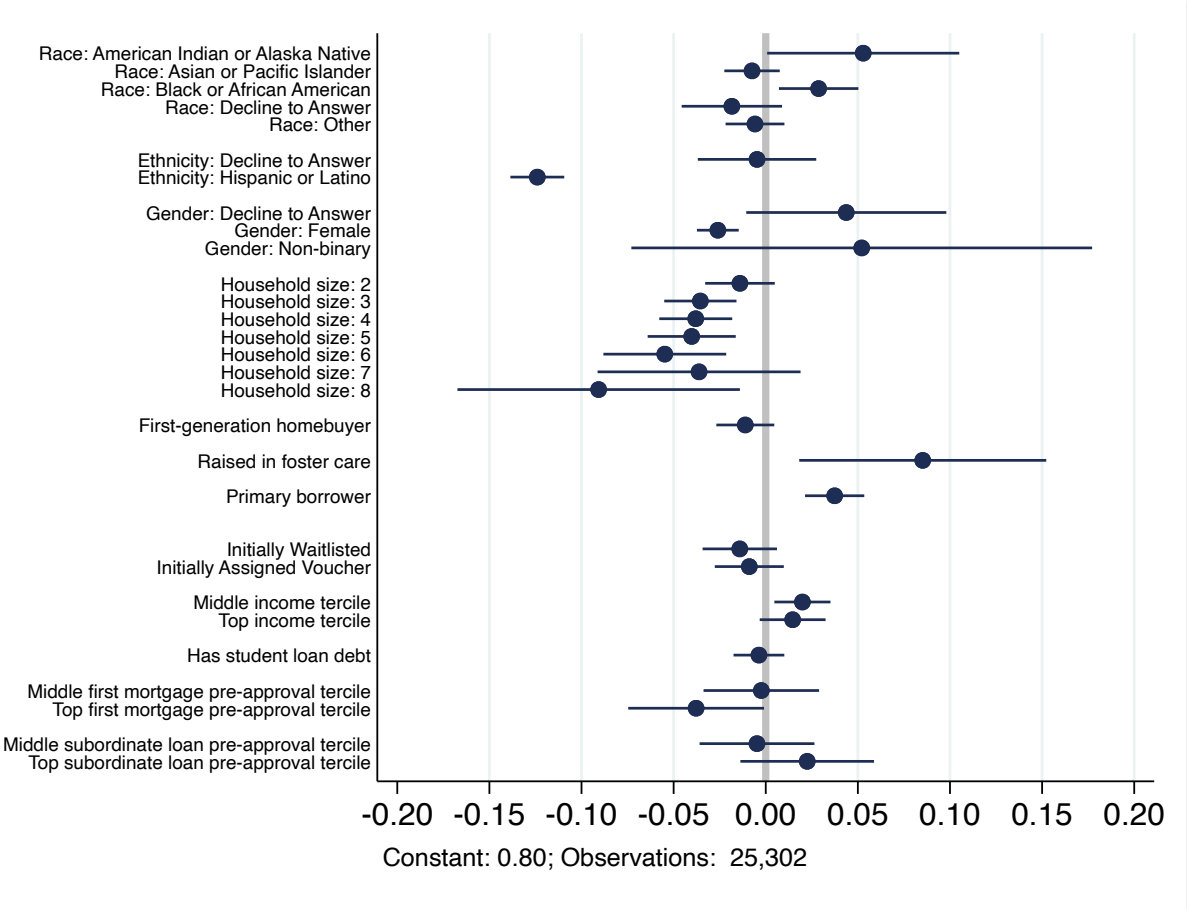
Notes: This table describes the linkage rounds we conducted. All CalHFA data had non-missing values for names and DOBs, so a perfect match on these fields implied that we dropped bad names or DOBs. For fuzzy merging names and dates of birth, we used the default scoring functions in Fu, Davis, Rothstein, Ramesh and White (2022) associated GitHub repository to assess string similarity. We defined a scoring function for addresses based on address numbers, direction codes (e.g., "West" in "West 5th Street"), street names, street suffixes (e.g., "Street" in "West 5th Street"), unit IDs (e.g., "Apt A"), city, state, and ZIP code. A "strict fuzzy match" exactly matches all these component fields except unit ID and street name suffix; exactly matches on house number, street name, city, state, and ZIP; exactly matches on house number, street name, city, and state; or exact matches on house number, city, and state, and Soundex matches or first four letter matches on street name. A "loose fuzzy match" includes the partial matches described above, or exact matches on city and state, plus exact match on house number or Soundex match or first four letters match on street name; exact match on state, plus two of exact match on house number, city match on Soundex, and Soundex match or first letter match on street name; and exact match on state, plus one of exact match on house number; city match on soundex; and soundex match or first letter match on street name.

Figure B.1: CalHFA-UCCCP hashed linkage algorithm results



Notes: This figure shows the overall match rate between CalHFA and UCCCP data, disaggregating by round, after de-duplication by CalHFA program ID. Criteria for each matching round are enumerated in Table B.1.

Figure B.2: Predictors of matching CalHFA data to UCCCP data



Notes: This figure shows coefficients and 95% confidence intervals (with robust standard errors) from a regression of an indicator for having matched to the UCCCP data. The regressors are from the CalHFA program data set, and the regression is estimated among all Dream for All applicants we observe in the program data set. The omitted race, ethnicity, gender, and household size categories are White, Not Hispanic or Latino, Male, and Household size: 1, respectively. The omitted lottery status category is "Not Selected."

more likely to be matched than those listed as secondary borrowers. The strongest predictor of a match, however, is the presence of an SSN in the CalHFA data: holding all else constant, having an SSN increases the likelihood of a successful match by about 15 percentage points. Controlling for the presence of an SSN, individuals who are initially waitlisted or assigned an offer are no more likely to be matched to UCCCP records than those who were not selected for an offer. However, the CalHFA data only has SSNs for individuals who used an offer to take out a Dream for All loan. Merge rates do not depend strongly on borrowers' income, student loan debt, or loan pre-approvals.

We conclude with three additional merge quality checks, which compare the values of comparable sets of variables across data sources and check whether individuals' characteristics in the UCCCP suggest that they could have qualified for a Dream for All loan. These exercises disaggregate statistics by hashed linkage round, comparing merges from earlier rounds – which impose more stringent criteria – to merges from later rounds. If earlier merges show much lower rates of data anomalies than merges in later rounds, this would suggest that later round merges are of much lower quality, and call for more careful evaluation of merging criteria. If, on the contrary, the number of data anomalies in later rounds is comparable to early round merges, this suggests that the overall linkage strategy is sound.

Table B.2: Comparing characteristics across data sources among merged individuals

	Percent in round				
	1	2	3	4	5
Panel A. All individuals					
Any open mortgage loans	0.4	0.5	0.5	0.2	0.5
Student loan mismatch	3.7	4.5	3.9	3.9	4.5
Gender mismatch	5.5	5.7	5.4	5.0	6.9
Different household size	87.5	89.2	86.7	89.5	87.6
Household size differs by ≥ 1	60.2	64.6	59.0	63.6	65.3
Household size differs by ≥ 2	35.8	40.5	36.1	41.1	38.1
Panel B. Individuals on same application in CalHFA, all merged to UC-CCP data					
Different household IDs	7.4	44.1	26.4	15.6	18.5
Different last names in CalHFA data	67.0	65.6	63.0	76.6	49.4
Different last names in UC-CCP data	67.0	65.6	63.0	69.1	48.1
Different addresses in UC-CCP data	6.8	46.2	26.0	14.8	16.0
Different addresses in CalHFA data	9.4	24.7	13.4	11.3	13.6

Notes: This table shows the results of diagnostic tests comparing values in CalHFA data fields to values in comparable UCCCP data fields. Panel A examines individual characteristics, whereas Panel B examines characteristics of sets of individuals associated with the same Dream for All pre-registration (i.e., with the same application).

Our first diagnostic test assesses whether merged individuals have any open mortgage loans according to 2024 first quarter UCCCP records. In order to apply for Dream for

All loans, applicants had to attest that they were first-time homebuyers, and their first-time homebuyer status was subject to a random quality assurance-quality control check by CalHFA. This means that Dream for All applicants were extremely unlikely to have open mortgages at the time they applied. The first row of Table B.2 shows that, indeed, a vanishingly small share of merged applicants have open mortgages according to UCCCP data.

Second, we use demographic variables reported in both the CalHFA pre-registrations and UCCCP data. Dream for All applicants were asked to report their gender and household size, as well as whether they had student loans, when submitting the pre-registration. The UCCCP infers gender using name evaluation software and household size using a private algorithm, and its tradeline data cover student loan borrowers. The second through sixth rows of Table B.2 show the shares of individuals for whom the student loan status, gender, and household size in the CalHFA data did not match to the corresponding variables in the UCCCP data. The gender and student loan mismatch shares are low and also reassuringly consistent across matching rounds. While household sizes often differ between data sources, they are not usually off by more than two individuals.

Our final set of tests restricts attention to individuals whose CalHFA applications included multiple individuals, more than one of whom was merged to a UCCCP record. "Round" here refers to the round in which the latest individual associated with the application ID was matched. The first row of Table B.2 under Panel B shows that most individuals who were on the same Dream for All application also share a household ID in the credit bureau data. While there is considerable variation in this statistic across rounds – with some more extreme values in rounds with few merges – there is no clear increasing pattern in this share across rounds. The remaining rows in Panel B show that similar shares of individuals associated with the same CalHFA application have the same last name or same address, regardless of whether we use the CalHFA or UCCCP data.

C Derivations and proofs

Notation and conventions Expectations are taken over the home price state and not the applicant population unless otherwise noted. We denote state-H and state-L consumption respectively as c_1^H and c_1^L, so $E[1/c_1] = \pi/c_1^H + (1-\pi)/c_1^L$.

For an owner financing fractions $\theta_{\text{cash}}, \theta_{\text{sal}}$ of a home purchase with cash and the SAL, respectively, define the per-unit user cost in state $j \in \{H, L\}$, in period-1 dollars, as

$$\omega_j \equiv (1+\rho)\theta_{\text{cash}}p_0 + (1+r(1-\theta))(1-\theta)p_0 + \theta_{\text{sal}}\left[p_0 + \gamma(p_j - p_0)^+\right] - p_j.$$

The expected user cost defined in the main text is $[\pi\omega_H + (1-\pi)\omega_L]/(1+\rho)$, so the assumption (A1) that the present value of rent does not exceed the minimized expected user cost can be written as $q \leq \pi\omega_H + (1-\pi)\omega_L$ at every financing plan. Let $s \equiv w - \theta_{\text{cash}}p_0 h - c_0 \geq 0$ denote a homebuyer's saving. Her period-1 budget constraint in state j is

$$c_1^j = (1+\rho)s + y - [\omega_j - (1+\rho)\theta_{\text{cash}}p_0]\, h = (1+\rho)(w - c_0) + y - \omega_j h,$$

where $\omega_j - (1+\rho)\theta_{\text{cash}}p_0$ is the part of the user cost she pays in period 1. We write the maximum expected utility from housing and non-housing consumption under renting, unassisted ownership, and SAL-assisted ownership as $v_R(w)$, $v_O(w)$ and $v_O^{\text{sal}}(w)$, respectively so that $V_R = v_R$, $V_O = v_O + \phi$, and $V_O^{\text{sal}} = v_O^{\text{sal}} + \phi - \kappa$.

For an SAL with appreciation-sharing parameter bounded by $\gamma < (p_H - p_L)/(p_H - p_0)$, we have, first, that

$$\omega_L - \omega_H = (p_H - p_L) - \gamma\theta_{\text{sal}}(p_H - p_0) > 0 \quad \Rightarrow \quad c_1^H - c_1^L = (\omega_L - \omega_H)h > 0,$$

so the user cost is higher, and period-1 consumption lower, in state L. Second, by A1 and the first fact, $\omega_L \geq \pi\omega_H + (1-\pi)\omega_L \geq q > 0$.

Finally, we introduce a convention for the value of a dollar saved in period 0, which returns $1+\rho$ dollars in period 1. Each of these $1+\rho$ dollars is worth the agent's marginal utility of consumption $(1-\beta)/c_1$, discounted by $(1+\rho)^{-1}$, so, in expectation, a dollar of saving is worth $(1-\beta)E[1/c_1]$. At any optimum, a dollar of period-0 consumption is worth at least as much as a dollar saved; that is, $(1-\beta)/c_0 \geq (1-\beta)E[1/c_1]$, with equality if the agent has positive saving. If the inequality is strict, she saves nothing. For an owner at her optimum, define her *shadow savings rate* $\hat{\rho}$ by

$$\frac{1}{c_0} = \frac{1+\hat{\rho}}{1+\rho}\, E\left[\frac{1}{c_1}\right],$$

the return on saving at which she would be exactly willing to save a first dollar. By the inequality above, $\hat{\rho} \geq \rho$, which holds with equality exactly when $1/c_0 = E[1/c_1]$, and in particular whenever she saves a positive amount.

Lemma 1 (Uniqueness, continuity, and the value of wealth)**.** *(a) The problem of the renter, unassisted owner, and SAL-assisted owner each has a unique solution at every wealth at which it is feasible, and the solution is continuous in wealth. The same is true of each problem modified by any combination of the following: the period-0 constraint deleted*

(so that saving s may be negative), the financing plan or any one of its shares held fixed, period-0 consumption fixed at w, or an upper bound placed on the SAL share, in which case the solution is also continuous in the bound.

(b) Each value function v is concave and continuous, and is differentiable with $v'(w) = (1-\beta)/c_0(w)$, where $c_0(w)$ is optimal period-0 consumption. Consequently $v(w_2) - v(w_1) = \int_{w_1}^{w_2} (1-\beta)/c_0(w)\,dw$ for any $w_1 < w_2$ in its domain.

Proof. We begin with (a). Consider the cash down payment $\theta_{\text{cash}} p_0 h$, the SAL principal $\theta_{\text{sal}} p_0 h$, and the mortgage balance $\ell = p_0 h - \theta_{\text{cash}} p_0 h - \theta_{\text{sal}} p_0 h$. In these variables, every constraint of every problem is linear in the choice variables and in w. The mortgage repayment is $(1 + r(\ell/p_0 h))\,\ell$. Define $f(x) \equiv (1 + r(x))x$, so that the mortgage repayment is $p_0 h\, f(\ell/p_0 h)$ and $f''(x) = 2r'(x) + xr''(x) > 0$.

Claim: The mortgage repayment function $(\ell, h) \mapsto p_0 h\, f(\ell/p_0 h)$ is jointly convex in (ℓ, h) on $h > 0$.

Proof of claim: We want to show that at any two points $(\ell_1, h_1), (\ell_2, h_2)$ with $h_1, h_2 > 0$ and any $t \in [0,1]$, the function evaluated at the weighted average $(t\ell_1 + (1-t)\ell_2,\ th_1 + (1-t)h_2)$ is at most the weighted average of its values at the two points. The weights $th_1/\left[th_1 + (1-t)h_2\right]$ and $(1-t)h_2/\left[th_1 + (1-t)h_2\right]$ are nonnegative and sum to one, so convexity of f gives

$$tp_0 h_1 f\left(\frac{\ell_1}{p_0 h_1}\right) + (1-t)p_0 h_2 f\left(\frac{\ell_2}{p_0 h_2}\right) \geq p_0\left[th_1 + (1-t)h_2\right] f\left(\frac{t\ell_1 + (1-t)\ell_2}{p_0\left[th_1 + (1-t)h_2\right]}\right),$$

which is the desired result. //

Now period-1 consumption in state j can be written in terms of the following: period-0 resources not consumed or put down in cash, $w - c_0 - \theta_{\text{cash}} p_0 h$, grown at $1+\rho$, plus income y and sale proceeds $p_j h$, less the mortgage repayment $p_0 h\, f(\ell/p_0 h)$ and the SAL repayment, which is the SAL principal times $1 + \gamma(p_j - p_0)^+/p_0$. Every term is linear in the choice variables c_0, h, the cash down payment, the SAL principal, and the mortgage balance except the mortgage repayment, which enters with a negative sign and is convex by the claim. For the renter, $c_1 = (1+\rho)(w - c_0) + y - qh$ is linear. So in each state c_1^j is a concave function of the choice variables.

The logarithm is concave and increasing, so $\ln c_1^j$ is concave, and $\beta \ln(h - \underline{h})$ and $(1-\beta)\ln c_0$ are strictly concave in h and c_0. So the objective is concave on a convex constraint set.

To prove that each problem has a *unique* solution, we show the objective is strictly concave in every direction, that is, along every nonzero vector of changes $(dc_0, dh, d\ell)$ in period-0 consumption, housing, and the mortgage balance, together with changes in the cash down payment and the SAL principal that sum to $p_0\,dh - d\ell$. Strict concavity along a direction means that if we restrict the objective to a line through any allocation in that direction, the resulting function of one variable (position along the line) is strictly concave. Any nonzero direction falls into one of three cases.

First, suppose the direction changes housing or period-0 consumption ($dh \neq 0$ or $dc_0 \neq 0$). Moving a distance τ along the line changes h to $h + \tau\,dh$, so $\beta \ln(h + \tau\,dh - \underline{h})$ has second derivative $-\beta\,dh^2/(h + \tau\,dh - \underline{h})^2 < 0$ in τ, and the same holds for $(1-\beta)\ln(c_0 + \tau\,dc_0)$. So at least one of these two terms is strictly concave along the line. The remaining term of the

objective, $E \ln c_1^j$, is concave along it, and a strictly concave function plus a concave function is strictly concave.

Second, suppose the direction leaves housing and period-0 consumption fixed but changes the mortgage balance ($dh = dc_0 = 0$ and $d\ell \neq 0$). Then $\beta \ln(h - \underline{h})$ and $(1-\beta)\ln c_0$ do not move, and the question is whether $E \ln c_1^j$ is strictly concave along the line. Moving a distance τ along the line changes the mortgage balance to $\ell + \tau\, d\ell$, so with h fixed the mortgage repayment is $p_0 h\, f\big((\ell + \tau\, d\ell)/p_0 h\big)$, whose second derivative in τ is $f''\big((\ell+\tau\, d\ell)/p_0 h\big)\, d\ell^2/p_0 h > 0$. The repayment enters c_1^j with a negative sign, and the cash and SAL amounts that absorb the change in ℓ enter linearly, so in each state the second derivative of c_1^j in τ is $-f''\big((\ell + \tau\, d\ell)/p_0 h\big)\, d\ell^2/p_0 h < 0$. Writing c_1^j for the value and $\dot{c}_1^j, \ddot{c}_1^j$ for its first two derivatives in τ, the second derivative of $\ln c_1^j$ in τ is $\ddot{c}_1^j/c_1^j - (\dot{c}_1^j)^2/(c_1^j)^2 < 0$, so $\ln c_1^j$, and hence $E \ln c_1^j$, is strictly concave along the line.

Third, suppose the direction leaves housing, period-0 consumption, and the mortgage balance all fixed ($dh = dc_0 = d\ell = 0$). Then the only thing that can move is the split of the fixed down payment $p_0 h - \ell$ between cash and SAL, and for the direction to be nonzero it must move; scale the direction so that moving a distance τ along the line shifts τ dollars from SAL to cash. Each dollar shifted takes a dollar out of period-0 resources that would otherwise have been saved, lowering period-1 consumption by $1+\rho$ in both states, and removes a dollar of SAL principal from repayment, raising period-1 consumption by $1+\gamma g^+$ in state H and by exactly one in state L. So moving a distance τ changes state-L consumption to $c_1^L - \rho\tau$, and $\ln(c_1^L - \rho\tau)$ has second derivative $-\rho^2/(c_1^L - \rho\tau)^2 < 0$ in τ. So $\ln c_1^L$ is strictly concave along the line, and since every other term of the objective is concave along it, so is the objective. We use state L rather than state H because state-H consumption changes to $c_1^H + (\gamma g^+ - \rho)\tau$, which is constant in τ at $\gamma = \gamma_{\text{fair}}$, whereas the coefficient $-\rho$ on τ in state L is never zero. Now suppose towards a contradiction that there are two distinct optima. Their midpoint is feasible, and by strict concavity along the direction connecting them it is strictly preferred to both, which contradicts. So the optimum is unique, and by the theorem of the maximum it is continuous in w, and in any upper bound placed on the SAL share. Each of the listed modifications adds or deletes linear constraints, so the constraint set remains convex and the same argument applies.

We note one further consequence of the objective's concavity, used in proofs below: the first-order conditions are sufficient as well as necessary for optimality. Because the objective is concave and the constraint set is convex, an allocation at which no feasible marginal change raises the objective (that is, one satisfying the first-order conditions) is the unique optimum.

Now we show (b). Let w_1, w_2 be in the domain of v and let $t \in [0,1]$. The convex combination, with weights t and $1-t$, of the optimal allocations at w_1 and w_2 is feasible at $tw_1 + (1-t)w_2$, because every constraint is linear in w and the choice variables, and by concavity of the objective it delivers at least $tv(w_1) + (1-t)v(w_2)$. So $v(tw_1 + (1-t)w_2) \geq tv(w_1) + (1-t)v(w_2)$, and v is concave. A concave function on an interval is continuous. Now fix w and let $\epsilon > 0$ be small. The optimal allocation at $w+\epsilon$ with period-0 consumption reduced by ϵ is feasible at w, and the optimal allocation at w with period-0 consumption raised by ϵ is feasible at $w + \epsilon$, so

$$(1-\beta)\left[\ln\left(c_0(w) + \epsilon\right) - \ln c_0(w)\right] \leq v(w+\epsilon) - v(w) \leq (1-\beta)\left[\ln c_0(w+\epsilon) - \ln\left(c_0(w+\epsilon) - \epsilon\right)\right].$$

Dividing by ϵ and letting $\epsilon \to 0$, using continuity of $c_0(\cdot)$ from (a), shows that the right derivative of v at w exists and equals $(1-\beta)/c_0(w)$; the same argument with $w-\epsilon$ in place of w gives the left derivative. So v is differentiable with continuous derivative $(1-\beta)/c_0(w)$, and the integral formula is the fundamental theorem of calculus. □

Proposition 1 (Fairly-priced SAL)**.** *Suppose $\gamma \geq \gamma_{fair} = \rho/(\pi g^+)$; that rents, prices, and the mortgage rate schedule are unaffected by SAL availability; that mortgage lenders have zero welfare weight; and that the government values SAL cash flows at expected value discounted at ρ. Then providing an SAL leaves every eligible agent weakly better off, and every agent for whom buying with the SAL is the unique best alternative strictly better off; and the government earns a nonnegative expected present value on every dollar it lends, exactly zero at $\gamma = \gamma_{fair}$. So, if some agents take up the SAL, the program is a Pareto improvement.*

Proof. First we show that no eligible agent is worse off. An agent with infinite hassle cost faces the same problem with and without the program. An agent with zero hassle cost can rent or buy unassisted exactly as before, and can in addition buy with the SAL. Every unassisted financing plan is a SAL financing plan with $\theta_{\text{sal}} = 0$, so $V_O^{\text{sal}} \geq V_O$, and her best indirect utility weakly rises; it strictly rises if buying with the SAL is her unique best alternative.

Now we show that the government does not lose in expectation. A dollar lent through the SAL in period 0 returns $1 + \gamma g^+$ dollars in state H — the principal plus the share γ of the appreciation $(p_H - p_0)/p_0$ per dollar of principal — and 1 dollar in state L. Its expected present value is therefore

$$\frac{\pi(1+\gamma g^+) + (1-\pi)}{1+\rho} = \frac{1+\pi\gamma g^+}{1+\rho} \geq 1 \quad \Leftrightarrow \quad \pi\gamma g^+ \geq \rho \quad \Leftrightarrow \quad \gamma \geq \gamma_{\text{fair}},$$

with equality throughout at $\gamma = \gamma_{\text{fair}}$. So no eligible agent loses, takers with a unique best alternative gain, and the government does not lose in expectation, which is what we wanted to show. □

Lemma 2 (The renter)**.** *Let $(1-\beta)(y - q\underline{h})/(1+\beta\rho)$ be the renter's saving threshold.*

(i) The renter saves if and only if her wealth exceeds the threshold. At or below it, her period-0 consumption c_0 is her entire wealth endowment, and her housing consumption is $\underline{h} + \beta(1+\rho)(y - q\underline{h})/\big(q(1+\beta\rho)\big)$, which does not depend on w.

(ii) Above it,

$$c_0 = \frac{(1-\beta)\left[(1+\rho)w + y - q\underline{h}\right]}{2+\rho-\beta}, \qquad c_1 = c_0, \qquad h = \underline{h} + \frac{\beta(1+\rho)\left[(1+\rho)w + y - q\underline{h}\right]}{q\,(2+\rho-\beta)}. \tag{6}$$

Equivalently, she divides her lifetime resources net of subsistence rent, $w + (y - q\underline{h})/(1+\rho)$, between housing beyond subsistence (at its present cost $q(h - \underline{h})/(1+\rho)$), period-0 consumption, and the present value of period-1 consumption in the proportions $\beta : (1-\beta) : (1-\beta)/(1+\rho)$.

(iii) The two expressions for optimal housing consumption agree at the threshold, and housing is continuous and nondecreasing in wealth.

Proof. Whether or not she saves, the renter's first-order condition for housing consumption is

$$\frac{\beta}{h-\underline{h}} = \frac{(1-\beta)\,q}{(1+\rho)\,c_1},$$

because a unit of housing costs q dollars of period-1 consumption, each worth $(1-\beta)/c_1$, discounted by $1/(1+\rho)$.

First we show (i). Suppose she consumes her entire endowment, so $c_0 = w$ and $c_1 = y - qh$. Then the first-order condition for housing is $\beta(1+\rho)(y-qh) = (1-\beta)q(h-\underline{h})$, and:

$$\begin{aligned}
\beta(1+\rho)y + (1-\beta)q\underline{h} = hq\left[\beta(1+\rho) + (1-\beta)\right] &= hq(1+\beta\rho) \\
\Rightarrow h = \frac{\beta(1+\rho)y + (1-\beta)q\underline{h}}{q(1+\beta\rho)} &= \underline{h} + \frac{\beta(1+\rho)(y-q\underline{h})}{q(1+\beta\rho)} \\
\Rightarrow c_1 = y - qh = \frac{(1+\beta\rho)y - \beta(1+\rho)y - (1-\beta)q\underline{h}}{1+\beta\rho} &= \frac{(1-\beta)(y-q\underline{h})}{1+\beta\rho}.
\end{aligned}$$

The display shows that if she saves nothing, her period-1 consumption is exactly the threshold. We now ask when saving nothing is in fact her best choice.

Consider the allocation just computed: zero saving, and the housing that solves the first-order condition for housing given zero saving. By the assumption that $c_0 = w$, she has only two remaining choices, housing and saving. Her housing already satisfies its first-order condition, so a small change in housing cannot raise her utility. Her saving is zero and cannot be negative, so the only other small change available to her is to save a dollar. By Lemma 1 (uniqueness of optima) the no-saving allocation is her optimum if and only if that first dollar of saving does not raise her utility.

Does it? From the conventions established at the beginning of this appendix, a dollar saved is worth $(1-\beta)E[1/c_1]$. The renter's period-1 consumption is certain, so this is $(1-\beta)/c_1$, where c_1 is her period-1 consumption at the no-saving allocation, which we just showed equals the threshold. The dollar comes out of period-0 consumption, which at the no-saving allocation is all of her wealth, so it costs $(1-\beta)/w$. The dollar of saving does not increase her utility when $(1-\beta)/w \geq (1-\beta)/c_1$, or, equivalently, when $w \leq c_1$.

So when her wealth is at or below the threshold, saving nothing is optimal, and her housing is the number computed above, which does not involve w. When her wealth is above the threshold, saving a first dollar strictly raises her utility, so consuming her entire wealth endowment is not optimal; and, because the housing first-order condition pins down a unique no-saving allocation, no other zero-saving allocation can be optimal either. So she saves a positive amount. This proves (i).

Now we show (ii). Again, by the conventions discussion at the start of this appendix, if the agent saves, a dollar saved is worth a dollar consumed, $1/c_0 = 1/c_1$, so $c_1 = c_0$. Then the first-order condition for housing becomes $q(h-\underline{h}) = \beta(1+\rho)c_0/(1-\beta)$. Her lifetime budget constraint is $c_0 + c_1/(1+\rho) = w + (y-qh)/(1+\rho)$. Substituting $c_1 = c_0$, moving the subsistence rent to the right side and the remaining rent to the left, and then substituting

the first-order condition for housing:

$$\begin{aligned} c_0 + \frac{c_0}{1+\rho} + \frac{q(h-\underline{h})}{1+\rho} &= w + \frac{y-q\underline{h}}{1+\rho} \\ \Rightarrow c_0 \left[1 + \frac{1}{1+\rho} + \frac{\beta}{1-\beta}\right] &= w + \frac{y-q\underline{h}}{1+\rho} \\ \Rightarrow c_0 \frac{2+\rho-\beta}{(1-\beta)(1+\rho)} &= \frac{(1+\rho)w + y - q\underline{h}}{1+\rho} \\ \Rightarrow c_0 &= \frac{(1-\beta)\left[(1+\rho)w + y - q\underline{h}\right]}{2+\rho-\beta}, \end{aligned}$$

which is the first formula in (6); substituting it into the first-order condition for housing gives the third. The three uses of resources are $q(h-\underline{h})/(1+\rho) = \beta c_0/(1-\beta)$, c_0, and $c_0/(1+\rho)$, which are in the proportions $\beta : (1-\beta) : (1-\beta)/(1+\rho)$.

Finally we show (iii). At the threshold $w = (1-\beta)(y-q\underline{h})/(1+\beta\rho)$,

$$(1+\rho)w + y - q\underline{h} = (y-q\underline{h})\,\frac{(1+\rho)(1-\beta)+1+\beta\rho}{1+\beta\rho} = (y-q\underline{h})\,\frac{2+\rho-\beta}{1+\beta\rho},$$

so the housing formula in (ii) evaluates to $\underline{h} + \beta(1+\rho)(y-q\underline{h})/\big(q(1+\beta\rho)\big)$, the constant in (i). Housing is constant in w below the threshold and increasing in w above it, so it is continuous and nondecreasing. □

Lemma 3 (User cost of unassisted ownership)**.** *Without the SAL, the expected user cost of a unit of housing at down-payment share θ_{cash} is*

$$\theta_{cash} p_0 + \frac{(1+r(1-\theta_{cash}))(1-\theta_{cash})p_0 - E[p_1]}{1+\rho}.$$

The marginal cost of mortgage financing, $r(1-\theta_{cash}) + r'(1-\theta_{cash})(1-\theta_{cash})$, is strictly decreasing in θ_{cash}, exceeds the savings rate ρ at $\theta_{cash} = \underline{\theta}$, and is below ρ at $\theta_{cash} = 1$, so equation (1) has a unique solution $\theta^*_{cash} \in (\underline{\theta}, 1)$. The expected user cost is strictly decreasing in θ_{cash} on $[\underline{\theta}, \theta^*_{cash}]$ and strictly increasing on $[\theta^*_{cash}, 1]$, so it is minimized at θ^*_{cash} and nowhere else. Under A1, an unassisted buyer with $\theta_{cash} < \theta^*_{cash}$ faces an expected user cost strictly above the present value of the rent, $q/(1+\rho)$.

Proof. Differentiating the expected user cost in θ_{cash}:

$$\begin{aligned} &\frac{d}{d\theta_{\text{cash}}}\left[\theta_{\text{cash}} p_0 + \frac{(1+r(1-\theta_{\text{cash}}))(1-\theta_{\text{cash}})p_0 - E[p_1]}{1+\rho}\right] \\ &\quad = p_0 - \frac{p_0\left[1 + r(1-\theta_{\text{cash}}) + r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})\right]}{1+\rho} \\ &\quad = \frac{p_0}{1+\rho}\Big\{\rho - \left[r(1-\theta_{\text{cash}}) + r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})\right]\Big\}. \end{aligned}$$

An extra dollar of down payment costs a dollar now and saves, in period 1, the interest on the marginal mortgage dollar plus the repricing of the whole balance. Now differentiate the

marginal cost of mortgage financing in θ_{cash}:

$$\frac{d}{d\theta_{\text{cash}}}\left[r(1-\theta_{\text{cash}})+r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})\right]=-2r'(1-\theta_{\text{cash}})-r''(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})<0,$$

because $r'>0$ and $r''\geq 0$. At $\theta_{\text{cash}}=\underline{\theta}$ the marginal cost is $r(1-\underline{\theta})+r'(1-\underline{\theta})(1-\underline{\theta})>r(1-\underline{\theta})\geq\rho$, and at $\theta_{\text{cash}}=1$ it is $r(0)<\rho$. A continuous, strictly decreasing function that starts above ρ and ends below it equals ρ exactly once, at $\theta^*_{\text{cash}}\in(\underline{\theta},1)$. By the first display, the expected user cost is strictly decreasing in θ_{cash} while the marginal cost is above ρ and strictly increasing once it is below.

Finally, let $\theta_{\text{cash}}<\theta^*_{\text{cash}}$. Then the expected user cost at θ_{cash} strictly exceeds its value at θ^*_{cash}, which is at least the minimum of the expected user cost over all financing plans, which is at least $q/(1+\rho)$ by A1. □

Lemma 4 (The down payment and the period-0 budget constraint)**.** *Consider the unassisted buyer's optimum at a wealth above $\underline{\theta}p_0\underline{h}$. She never puts the full price down: $\theta_{cash}<1$. Her down-payment share equals the cost-minimizing share θ^*_{cash} if and only if the period-0 constraint carries no shadow value, which is the case exactly when a dollar of period-0 consumption is worth the same as a dollar saved, $1/c_0=E[1/c_1]$; this holds whenever the constraint is slack. If instead the constraint binds with positive shadow value, $1/c_0>E[1/c_1]$, her share is strictly below θ^*_{cash}, and the marginal cost of mortgage financing at her share strictly exceeds ρ. In particular, she chooses the minimum share $\underline{\theta}$ only when the constraint binds with positive shadow value.*

Proof. Consider raising the down-payment share by $d\theta$ at fixed h. This takes $p_0h\,d\theta$ dollars out of period-0 resources, and reduces the period-1 mortgage repayment in both states by $p_0h\,d\theta$ times $1+r(1-\theta_{\text{cash}})+r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})$. By the conventions discussion at the beginning of this appendix, the period-0 dollars come out of saving if she saves and out of consumption if she does not, as period-0 consumption is worth at least as much as savings. In either case, each is worth $(1-\beta)/c_0$. Each period-1 dollar is worth $(1-\beta)E[1/c_1]/(1+\rho)$. So the derivative of lifetime expected utility with respect to θ_{cash}, holding h fixed, is $(1-\beta)p_0h$ times

$$\begin{aligned}
&\frac{1+r(1-\theta_{\text{cash}})+r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})}{1+\rho}\,E\left[\frac{1}{c_1}\right]-\frac{1}{c_0}\\
&\quad=\frac{1+r(1-\theta_{\text{cash}})+r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})}{1+\rho}\,E\left[\frac{1}{c_1}\right]-\frac{1+\hat\rho}{1+\rho}\,E\left[\frac{1}{c_1}\right]\\
&\quad=\frac{E[1/c_1]}{1+\rho}\Big\{\left[r(1-\theta_{\text{cash}})+r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})\right]-\hat\rho\Big\},
\end{aligned}$$

where the equality substitutes the definition of the shadow savings rate $\hat\rho$. At an interior share the derivative is zero; at $\theta_{\text{cash}}=\underline{\theta}$ it is at most zero; and at $\theta_{\text{cash}}=1$ it would have to be at least zero. Recall that $\hat\rho\geq\rho$, with equality exactly when $1/c_0=E[1/c_1]$, that is, when the period-0 constraint carries no shadow value.

This derivative is signed by the difference between the marginal cost of mortgage financing at her chosen share and her shadow savings rate $\hat\rho$. There are three possibilities for her share: interior, at the upper bound 1, or at the lower bound $\underline{\theta}$.

First suppose the share is interior. Then the derivative of lifetime expected utility with respect to θ_{cash} must be zero, so the marginal cost of mortgage financing at her share equals $\hat{\rho}$. Recall from Lemma 3 that the marginal cost of mortgage financing is strictly decreasing in θ_{cash} and crosses ρ exactly once, at θ^*_{cash}. If $\hat{\rho} = \rho$, the marginal cost at her share equals ρ, so her share is θ^*_{cash}. If $\hat{\rho} > \rho$, the marginal cost at her share exceeds ρ, and since the marginal cost is decreasing this puts her share strictly below θ^*_{cash}.

Next suppose towards a contradiction that her share is 1, so that she borrows nothing. The share cannot be raised, so optimality requires only that the derivative of lifetime expected utility with respect to θ_{cash} is at least zero: the marginal cost of mortgage financing at a share of 1, which is $r(0)$, is at least $\hat{\rho}$. But $\hat{\rho} \geq \rho > r(0)$, which contradicts. So she always borrows something.

Finally suppose her share is the minimum $\underline{\theta}$. The share cannot be lowered, so optimality requires only that raising it does not pay, that is, that the derivative of lifetime expected utility with respect to θ_{cash} is at most zero: the marginal cost of mortgage financing at $\underline{\theta}$ is at most $\hat{\rho}$. By Lemma 3 that marginal cost exceeds ρ, so $\hat{\rho} > \rho$, which by the conventions above means $1/c_0 > E[1/c_1]$: the period-0 constraint binds with positive shadow value.

It remains to connect the shadow value to slackness. If the constraint is slack, she saves a positive amount, so by the conventions above $1/c_0 = E[1/c_1]$ and $\hat{\rho} = \rho$, and the interior case puts her share at θ^*_{cash}. This completes the proof. □

The following lemma writes an owner's optimal allocation in the form of the saving renter's (6). It is used only to compare an owner's period-0 consumption and housing with a renter's at the same wealth, in Proposition 2 and Proposition 3.

Lemma 5 (The owner's allocation in closed form)**.** *Consider the solution of either owner's problem at some wealth, with its financing plan, its state user costs ω_H, ω_L, and its shadow savings rate $\hat{\rho}$. Define her* effective rent *u as the marginal-utility-weighted average of her state user costs, with the cash down payment charged at the shadow rate $\hat{\rho}$ rather than at ρ:*

$$u \equiv \frac{\pi\big(\omega_H + (\hat{\rho} - \rho)\theta_{cash} p_0\big)/c_1^H + (1-\pi)\big(\omega_L + (\hat{\rho} - \rho)\theta_{cash} p_0\big)/c_1^L}{\pi/c_1^H + (1-\pi)/c_1^L}. \tag{7}$$

Then

$$c_0 = \frac{(1-\beta)(1+\rho)\left[(1+\hat{\rho})w + y - u\underline{h}\right]}{(1+\hat{\rho})(2+\rho-\beta)}, \qquad h = \underline{h} + \frac{\beta(1+\rho)\left[(1+\hat{\rho})w + y - u\underline{h}\right]}{u\,(2+\rho-\beta)}, \tag{8}$$

and $u > q$. In particular, when she saves ($\hat{\rho} = \rho$), (8) is the saving renter's allocation (6) with the rent q replaced by u.

Proof. We hold the financing plan fixed at its optimal value throughout, so that the only choices in play are h and c_0. We proceed in three steps: first we derive the first-order condition for housing, then we combine the two state budget constraints into one equation in c_0 and h, and finally we solve the two resulting equations for c_0 and h and show $u > q$.

First we derive the first-order condition for housing. At a fixed financing plan, a unit of housing costs the owner $\theta_{\text{cash}} p_0$ dollars in period 0 and, from the period-1 budget constraint $c_1^j = (1+\rho)(w - c_0) + y - \omega_j h$ written at the beginning of this appendix, $\omega_j - (1+\rho)\theta_{\text{cash}} p_0$

dollars in period 1 in state j. By the conventions discussion at the start of this appendix, the period-0 dollars come out of saving if she saves and out of consumption if she does not, and in either case each is worth $(1-\beta)/c_0$. Each period-1 dollar in state j is worth $(1-\beta)/c_1^j$, discounted by $1/(1+\rho)$. Housing is not constrained from above, and $h > \underline{h}$ at any optimum, so the derivative of lifetime expected utility with respect to h, holding the financing plan fixed, is zero at her optimum:

$$\begin{aligned}
\frac{\beta}{h-\underline{h}} &= (1-\beta)\left[\frac{\theta_{\text{cash}}p_0}{c_0} + \frac{1}{1+\rho}E\left[\frac{\omega_j - (1+\rho)\theta_{\text{cash}}p_0}{c_1^j}\right]\right] \\
&= (1-\beta)\left[\theta_{\text{cash}}p_0\,\frac{1+\hat{\rho}}{1+\rho}\,E\left[\frac{1}{c_1}\right] + \frac{1}{1+\rho}E\left[\frac{\omega_j - (1+\rho)\theta_{\text{cash}}p_0}{c_1^j}\right]\right] \\
&= \frac{1-\beta}{1+\rho}\,E\left[\frac{(1+\hat{\rho})\theta_{\text{cash}}p_0 + \omega_j - (1+\rho)\theta_{\text{cash}}p_0}{c_1^j}\right] \\
&= \frac{1-\beta}{1+\rho}\,E\left[\frac{\omega_j + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0}{c_1^j}\right],
\end{aligned}$$

where the second line substitutes the definition of the shadow savings rate, $1/c_0 = (1+\hat{\rho})E[1/c_1]/(1+\rho)$, and the third line uses $E[1/c_1] = \pi/c_1^H + (1-\pi)/c_1^L$ to bring the first term inside the expectation. Now, multiplying both sides of (7) by its denominator,

$$u\left[\frac{\pi}{c_1^H} + \frac{1-\pi}{c_1^L}\right] = \frac{\pi\big(\omega_H + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0\big)}{c_1^H} + \frac{(1-\pi)\big(\omega_L + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0\big)}{c_1^L},$$

$$\text{that is,} \qquad u\,E\left[\frac{1}{c_1}\right] = E\left[\frac{\omega_j + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0}{c_1^j}\right].$$

Substituting this into the last line of the previous display, and then substituting the definition of the shadow savings rate once more in the form $E[1/c_1] = (1+\rho)/\big((1+\hat{\rho})c_0\big)$,

$$\frac{\beta}{h-\underline{h}} = \frac{1-\beta}{1+\rho}\,u\,E\left[\frac{1}{c_1}\right] = \frac{1-\beta}{1+\rho}\cdot u\cdot\frac{1+\rho}{(1+\hat{\rho})\,c_0} = \frac{(1-\beta)\,u}{(1+\hat{\rho})\,c_0}.$$

Rearranging,

$$h - \underline{h} = \frac{(1+\hat{\rho})\,\beta\,c_0}{(1-\beta)\,u}. \tag{9}$$

Next we combine the two state budget constraints. In state j, $c_1^j = (1+\rho)(w-c_0)+y-\omega_j h$. We claim that this can be rewritten as

$$c_1^j = (1+\hat{\rho})(w-c_0) + y - \big(\omega_j + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0\big)h.$$

There are two cases. If she saves, then $\hat{\rho} = \rho$ by the conventions discussion, and the claimed expression is the budget constraint itself. If she does not save, then her period-0 resources

are exhausted by consumption and the cash down payment, $w - c_0 = \theta_{\text{cash}} p_0 h$, so

$$\begin{aligned}(1+\hat{\rho})(w-c_0) - (\hat{\rho}-\rho)\theta_{\text{cash}}p_0 h &= (1+\hat{\rho})\theta_{\text{cash}}p_0 h - (\hat{\rho}-\rho)\theta_{\text{cash}}p_0 h \\ &= (1+\rho)\theta_{\text{cash}}p_0 h = (1+\rho)(w-c_0),\end{aligned}$$

and substituting the left side for $(1+\rho)(w-c_0)$ in the budget constraint gives the claim. Now divide the claimed expression in state j by c_1^j, so that the left side is 1 in each state; multiply the state-H equation by π and the state-L equation by $1-\pi$; and add. The left side is $\pi + (1-\pi) = 1$, and the right side is

$$\begin{aligned}1 &= [(1+\hat{\rho})(w-c_0) + y]\left[\frac{\pi}{c_1^H} + \frac{1-\pi}{c_1^L}\right] \\ &\quad - h\left[\frac{\pi\big(\omega_H + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0\big)}{c_1^H} + \frac{(1-\pi)\big(\omega_L + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0\big)}{c_1^L}\right] \\ &= [(1+\hat{\rho})(w-c_0) + y]\,E\left[\frac{1}{c_1}\right] - h\,u\,E\left[\frac{1}{c_1}\right] \\ &= [(1+\hat{\rho})(w-c_0) + y - uh]\,E\left[\frac{1}{c_1}\right],\end{aligned}$$

where the second line uses the rearranged form of (7) displayed above. Dividing by $E[1/c_1]$ and using the definition of the shadow savings rate in the form $1/E[1/c_1] = (1+\hat{\rho})c_0/(1+\rho)$,

$$(1+\hat{\rho})(w-c_0) + y - uh = \frac{1}{E[1/c_1]} = \frac{(1+\hat{\rho})\,c_0}{1+\rho} \quad \Rightarrow \quad uh = (1+\hat{\rho})(w-c_0) + y - \frac{(1+\hat{\rho})\,c_0}{1+\rho}.$$

Finally we solve. Multiplying (9) by u gives $uh = u\underline{h} + (1+\hat{\rho})\beta c_0/(1-\beta)$. Substituting this for uh in the last display and moving every term in c_0 to the left:

$$\begin{aligned}u\underline{h} + \frac{(1+\hat{\rho})\,\beta\,c_0}{1-\beta} &= (1+\hat{\rho})w - (1+\hat{\rho})c_0 + y - \frac{(1+\hat{\rho})\,c_0}{1+\rho} \\ \Rightarrow (1+\hat{\rho})\,c_0\left[\frac{\beta}{1-\beta} + 1 + \frac{1}{1+\rho}\right] &= (1+\hat{\rho})w + y - u\underline{h}.\end{aligned}$$

Putting the bracket over the common denominator $(1-\beta)(1+\rho)$,

$$\frac{\beta}{1-\beta} + 1 + \frac{1}{1+\rho} = \frac{\beta(1+\rho) + (1-\beta)(1+\rho) + (1-\beta)}{(1-\beta)(1+\rho)} = \frac{(1+\rho) + (1-\beta)}{(1-\beta)(1+\rho)} = \frac{2+\rho-\beta}{(1-\beta)(1+\rho)},$$

so

$$(1+\hat{\rho})\,c_0\,\frac{2+\rho-\beta}{(1-\beta)(1+\rho)} = (1+\hat{\rho})w + y - u\underline{h} \quad \Rightarrow \quad c_0 = \frac{(1-\beta)(1+\rho)\,[(1+\hat{\rho})w + y - u\underline{h}]}{(1+\hat{\rho})(2+\rho-\beta)},$$

which is the first formula in (8). Substituting it into (9), the factor $(1+\hat{\rho})$ cancels:

$$h - \underline{h} = \frac{(1+\hat{\rho})\,\beta}{(1-\beta)\,u} \cdot \frac{(1-\beta)(1+\rho)\,[(1+\hat{\rho})w + y - u\underline{h}]}{(1+\hat{\rho})(2+\rho-\beta)} = \frac{\beta(1+\rho)\,[(1+\hat{\rho})w + y - u\underline{h}]}{u\,(2+\rho-\beta)},$$

which is the second formula in (8). When she saves, $\hat{\rho} = \rho$, and (8) becomes $c_0 = (1-\beta)\,[(1+\rho)w + y - u\underline{h}]\,/(2+\rho-\beta)$ and $h = \underline{h} + \beta(1+\rho)\,[(1+\rho)w + y - u\underline{h}]\,/\big(u(2+\rho-\beta)\big)$, which is (6) with u in place of q.

It remains to show $u > q$. The term $(\hat{\rho}-\rho)\theta_{\text{cash}}p_0$ appears in both adjusted user costs in (7), with weights π/c_1^H and $(1-\pi)/c_1^L$ that sum to the denominator, so it contributes exactly $(\hat{\rho}-\rho)\theta_{\text{cash}}p_0$ to u:

$$u = \frac{\pi\omega_H/c_1^H + (1-\pi)\omega_L/c_1^L}{\pi/c_1^H + (1-\pi)/c_1^L} + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0.$$

We compare the first term to the plain average $\pi\omega_H + (1-\pi)\omega_L$. Putting both over the common denominator $\pi/c_1^H + (1-\pi)/c_1^L$, the numerator of the difference is

$$\begin{aligned}
&\frac{\pi\omega_H}{c_1^H} + \frac{(1-\pi)\omega_L}{c_1^L} - [\pi\omega_H + (1-\pi)\omega_L]\left[\frac{\pi}{c_1^H} + \frac{1-\pi}{c_1^L}\right] \\
&\quad= \frac{\pi\omega_H}{c_1^H} + \frac{(1-\pi)\omega_L}{c_1^L} - \frac{\pi^2\omega_H}{c_1^H} - \frac{\pi(1-\pi)\omega_H}{c_1^L} - \frac{\pi(1-\pi)\omega_L}{c_1^H} - \frac{(1-\pi)^2\omega_L}{c_1^L} \\
&\quad= \frac{\pi(1-\pi)\omega_H}{c_1^H} - \frac{\pi(1-\pi)\omega_H}{c_1^L} + \frac{\pi(1-\pi)\omega_L}{c_1^L} - \frac{\pi(1-\pi)\omega_L}{c_1^H} \\
&\quad= \pi(1-\pi)\left[\omega_H\left(\frac{1}{c_1^H} - \frac{1}{c_1^L}\right) + \omega_L\left(\frac{1}{c_1^L} - \frac{1}{c_1^H}\right)\right] = \pi(1-\pi)\,(\omega_L - \omega_H)\left(\frac{1}{c_1^L} - \frac{1}{c_1^H}\right),
\end{aligned}$$

where the third line uses $\pi - \pi^2 = \pi(1-\pi)$ and $(1-\pi) - (1-\pi)^2 = \pi(1-\pi)$. From the beginning of this appendix, $\omega_L - \omega_H > 0$ and $c_1^L < c_1^H$, so $1/c_1^L - 1/c_1^H > 0$, and the numerator is strictly positive; the denominator $\pi/c_1^H + (1-\pi)/c_1^L$ is positive as well. So

$$\begin{aligned}
u = \frac{\pi\omega_H/c_1^H + (1-\pi)\omega_L/c_1^L}{\pi/c_1^H + (1-\pi)/c_1^L} + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0 &> \pi\omega_H + (1-\pi)\omega_L + (\hat{\rho}-\rho)\theta_{\text{cash}}p_0 \\
&\geq \pi\omega_H + (1-\pi)\omega_L \geq q,
\end{aligned}$$

where the second inequality uses $\hat{\rho} \geq \rho$ and $\theta_{\text{cash}}p_0 \geq 0$, and the third is A1 as written at the beginning of this appendix. This is what we wanted to show. □

Proposition 2 (The applicant population)**.** *Suppose A1 and A2 hold and* $\gamma < (p_H - p_L)/(p_H - p_0)$.

(i) The preference for homeownership at which an agent is indifferent between renting and buying unassisted, $v_R(w) - v_O(w)$*, is continuous and strictly decreasing in* w *on* $(\underline{\theta}p_0\underline{h}, \infty)$*, and tends to* $+\infty$ *as* w *falls to* $\underline{\theta}p_0\underline{h}$*.*

(ii) There is a wealth $\hat{w} > 0$ *such that an agent buying with the SAL consumes her entire endowment in period 0 and makes no cash down payment exactly when* $w \leq \hat{w}$*; on* $(0, \hat{w}]$

her housing and SAL share do not depend on w. The preference at which an agent with zero hassle cost is indifferent between renting and buying with the SAL, $v_R(w) - v_O^{sal}(w)$, is constant on $(0, \hat{w}]$ and continuous and strictly decreasing on $[\hat{w}, \infty)$.

(iii) $v_R(w) - v_O^{sal}(w) \leq v_R(w) - v_O(w)$ *at every* $w > \underline{\theta} p_0 \underline{h}$.

(iv) An agent with zero hassle cost buys with the SAL if and only if $\phi \geq v_R(w) - v_O^{sal}(w)$. *She is a marginal buyer if in addition* $w \leq \underline{\theta} p_0 \underline{h}$ *or* $\phi < v_R(w) - v_O(w)$, *and an inframarginal taker if* $w > \underline{\theta} p_0 \underline{h}$ *and* $\phi \geq v_R(w) - v_O(w)$; *otherwise she is a renter never-taker. An agent with infinite hassle cost buys unassisted if and only if* $w > \underline{\theta} p_0 \underline{h}$ *and* $\phi \geq v_R(w) - v_O(w)$ *(a buyer never-taker), and rents otherwise (a renter never-taker). The effect of SAL availability on the homeownership rate equals the population share of marginal buyers.*

Proof. Throughout, we write c_0^R, c_0^O, and c_0^{sal} for the renter's, the unassisted buyer's, and the SAL buyer's optimal period-0 consumption at a given wealth.

First we show (i). By Lemma 1(b), v_R and v_O are continuous and differentiable on $(\underline{\theta} p_0 \underline{h}, \infty)$ with $v_R'(w) = (1-\beta)/c_0^R$ and $v_O'(w) = (1-\beta)/c_0^O$, so their difference is continuous and differentiable with

$$\frac{d}{dw}\left[v_R(w) - v_O(w)\right] = (1-\beta)\left[\frac{1}{c_0^R} - \frac{1}{c_0^O}\right],$$
$$\left[v_R(w_2) - v_O(w_2)\right] - \left[v_R(w_1) - v_O(w_1)\right] = \int_{w_1}^{w_2} (1-\beta)\left[\frac{1}{c_0^R} - \frac{1}{c_0^O}\right] dw.$$

If $c_0^O < c_0^R$ at every $w > \underline{\theta} p_0 \underline{h}$, then $1/c_0^O > 1/c_0^R$, the integrand is strictly negative, and the integral is strictly negative for every $w_1 < w_2$; that is, $v_R - v_O$ is strictly decreasing. So it suffices to show $c_0^O < c_0^R$ at every $w > \underline{\theta} p_0 \underline{h}$. There are two cases, according to whether the renter saves.

Suppose the renter is at or below her saving threshold. Then $c_0^R = w$ by Lemma 2(i). The unassisted buyer must fund both consumption and her cash down payment out of w, so $c_0^O \leq w - \theta_{\text{cash}} p_0 h$; and since $\theta_{\text{cash}} \geq \underline{\theta}$ and $h > \underline{h}$, this gives $c_0^O \leq w - \theta_{\text{cash}} p_0 h < w - \underline{\theta} p_0 \underline{h} < w = c_0^R$.

Suppose instead the renter is above her saving threshold. Then c_0^R is given by (6), and c_0^O by (8) with the unassisted buyer's own effective rent u and shadow savings rate $\hat{\rho}$. Distributing the factor $(1+\rho)/(1+\hat{\rho})$ in (8) over the bracket, so that the term in w matches the renter's,

$$c_0^R = \frac{1-\beta}{2+\rho-\beta}\left[(1+\rho)w + \left(y - q\underline{h}\right)\right],$$
$$c_0^O = \frac{1-\beta}{2+\rho-\beta}\left[\frac{(1+\rho)(1+\hat{\rho})w}{1+\hat{\rho}} + \frac{1+\rho}{1+\hat{\rho}}\left(y - u\underline{h}\right)\right] = \frac{1-\beta}{2+\rho-\beta}\left[(1+\rho)w + \frac{1+\rho}{1+\hat{\rho}}\left(y - u\underline{h}\right)\right].$$

The two expressions share the factor in front and the term $(1+\rho)w$, so $c_0^O < c_0^R$ if and only if the buyer's second term is strictly below the renter's, $y - q\underline{h}$, which is strictly positive by A2. If $y - u\underline{h} \leq 0$, the buyer's second term is at most zero, and so strictly below $y - q\underline{h} > 0$.

If $y - u\underline{h} > 0$, then, because $\hat{\rho} \geq \rho$ gives $(1+\rho)/(1+\hat{\rho}) \leq 1$, the buyer's second term is at most $y - u\underline{h}$; and $y - u\underline{h} < y - q\underline{h}$ because $u > q$ (Lemma 5) and $\underline{h} > 0$. In either case $c_0^O < c_0^R$. This proves that $v_R - v_O$ is continuous and strictly decreasing.

Now we show the divergence. Fix any wealth w in $(\underline{\theta}p_0\underline{h}, 2\underline{\theta}p_0\underline{h}]$ and any feasible unassisted allocation at w. The period-0 constraint gives $c_0 \leq w - \theta_{\text{cash}}p_0h \leq w - \underline{\theta}p_0\underline{h}$, and it also gives $\underline{\theta}p_0h \leq \theta_{\text{cash}}p_0h \leq w$, so $h \leq w/(\underline{\theta}p_0) \leq 2\underline{h}$. Period-1 consumption in state j is period-0 resources not consumed or put down, grown at $1+\rho$, plus income and sale proceeds, less the mortgage repayment, so $c_1^j \leq (1+\rho)w + y + p_Hh \leq (1+\rho)w + y + 2p_H\underline{h}$. So every feasible allocation at w has

$$E\left[U(c_0, c_1, h)\right] \leq \beta \ln \underline{h} + (1-\beta)\ln\left(w - \underline{\theta}p_0\underline{h}\right) + \frac{1-\beta}{1+\rho}\ln\left[(1+\rho)w + y + 2p_H\underline{h}\right],$$

and so does $v_O(w)$, which is the maximum over such allocations. As w falls to $\underline{\theta}p_0\underline{h}$, the first and third terms on the right stay bounded while the second tends to $-\infty$, so $v_O(w) \to -\infty$. On the other hand, renting is feasible at every $w > 0$, since by A2 the renter can consume $c_0 = w$ and a house slightly above $\underline{h}$ and still have $c_1 = y - qh > 0$; so $v_R(w)$ is finite at every $w > 0$ and, being continuous by Lemma 1(b), stays bounded as w falls to $\underline{\theta}p_0\underline{h}$. So $v_R(w) - v_O(w) \to +\infty$, which proves (i).

Now we show (ii). We begin by defining the *wealth-free problem*: choose housing $h > \underline{h}$ and a SAL share $\theta_{\text{sal}} \in [\underline{\theta}, 1]$ to maximize

$$\beta \ln(h - \underline{h}) + \frac{1-\beta}{1+\rho}\, E \ln\left(y - \omega_j h\right),$$

where ω_j is the state-j user cost at the financing plan $(\theta_{\text{cash}}, \theta_{\text{sal}}) = (0, \theta_{\text{sal}})$. This is the SAL buyer's problem with period-0 consumption fixed at w and the cash share fixed at zero, up to the additive term $(1-\beta)\ln w$: with $c_0 = w$ and $\theta_{\text{cash}} = 0$ she saves nothing, so her period-1 consumption in state j is $y - \omega_j h$. It is feasible: at the plan $(0, \underline{\theta})$, $\omega_L = (1+r(1-\underline{\theta}))(1-\underline{\theta})p_0 + \underline{\theta}p_0 - p_L = (1+r(1-\underline{\theta}))p_0 - \underline{\theta}\, r(1-\underline{\theta})p_0 - p_L \leq (1+r(1-\underline{\theta}))p_0 - p_L$, so by the second part of A2 $y - \omega_L\underline{h} > 0$, and a house slightly above $\underline{h}$ leaves period-1 consumption positive in both states. By Lemma 1(a) (period-0 consumption fixed, one share held fixed) it has a unique solution. Neither the objective nor the constraints involve w, so the solution does not depend on w. We write c_1^H, c_1^L for the period-1 consumptions it delivers, and refer to the SAL buyer's allocation with $c_0 = w$, $\theta_{\text{cash}} = 0$, zero saving, and the wealth-free housing and SAL share as the *wealth-free allocation* at w.

We now ask at which wealths the wealth-free allocation is the SAL buyer's optimum. By Lemma 1 (sufficiency of the first-order conditions), it is her optimum at w if and only if no feasible marginal change raises her lifetime expected utility. At the wealth-free allocation the period-0 constraint $c_0 + \theta_{\text{cash}}p_0h \leq w$ holds with equality and the cash share is at its lower bound of zero, so the feasible marginal changes are: changes in h; changes in θ_{sal} at fixed cash, that is, exchanges of SAL for mortgage; moving a period-0 dollar out of consumption into saving; moving a period-0 dollar out of consumption into a cash down payment in place of a dollar of SAL principal, at a fixed mortgage; and moving a period-0 dollar out of consumption into a cash down payment on top of the SAL, in place of a dollar of mortgage. Every feasible marginal change is a combination of these with nonnegative weights, and the

derivative of lifetime expected utility along a combination is the same combination of the derivatives along its parts, so it suffices to check each in turn.

Changes in h, and exchanges of SAL for mortgage, do not raise her utility, because the wealth-free housing and SAL share satisfy the wealth-free problem's own first-order conditions, and the wealth-free problem's objective differs from hers only by the constant $(1-\beta)\ln w$. Moving a period-0 dollar into saving costs a dollar of period-0 consumption, worth $(1-\beta)/w$ since $c_0 = w$, and, by the conventions discussion at the start of this appendix, is worth $(1-\beta)E[1/c_1]$. Moving a period-0 dollar into a cash down payment in place of a dollar of SAL principal also costs $(1-\beta)/w$; it removes a dollar of SAL principal from repayment, and so raises period-1 consumption by $1+\gamma g^+$ in state H — the principal plus the share γ of the appreciation $(p_H - p_0)/p_0$ per dollar of principal — and by exactly one in state L, each period-1 dollar in state j being worth $(1-\beta)/c_1^j$ discounted by $1/(1+\rho)$; so it is worth $(1-\beta)\left[\pi(1+\gamma g^+)/c_1^H + (1-\pi)/c_1^L\right]/(1+\rho)$. The last change, a cash dollar on top of the SAL in place of a mortgage dollar, is the previous change combined with an exchange of mortgage for SAL, that is, an increase in the SAL share; the derivative along it is the sum of the derivatives along the two, and the second is at most zero at the wealth-free solution (it is zero if the SAL share is interior, at most zero if the share is at $\underline{\theta}$, and the change is infeasible if the share is 1). So the last change raises utility only if the previous one does, and need not be checked separately. Hence no feasible marginal change raises her utility exactly when

$$\frac{1-\beta}{w} \geq (1-\beta)E\left[\frac{1}{c_1}\right] \qquad \text{and} \qquad \frac{1-\beta}{w} \geq \frac{1-\beta}{1+\rho}\left[\frac{\pi(1+\gamma g^+)}{c_1^H} + \frac{1-\pi}{c_1^L}\right],$$

with c_1^H, c_1^L the wealth-free period-1 consumptions. Define $\hat{w}$ as the reciprocal of the larger of

$$E\left[\frac{1}{c_1}\right] \qquad \text{and} \qquad \frac{1}{1+\rho}\left[\frac{\pi(1+\gamma g^+)}{c_1^H} + \frac{1-\pi}{c_1^L}\right],$$

evaluated at the wealth-free solution; $\hat{w}$ does not depend on w, and $\hat{w} > 0$. Both inequalities in the display hold exactly when $1/w$ is at least the larger of the two numbers, that is, when $w \leq \hat{w}$. So for $w \leq \hat{w}$ the wealth-free allocation is the SAL buyer's optimum: she consumes her entire endowment, makes no cash down payment, and her housing and SAL share are the wealth-free solution, which does not depend on w. For $w > \hat{w}$ the wealth-free allocation is not her optimum. Moreover, any optimum at which she makes no cash down payment and saves nothing has $c_0 = w$ and $\theta_{\text{cash}} = 0$, so its housing and SAL share solve the wealth-free problem, and it is the wealth-free allocation; so at $w > \hat{w}$ her optimum has either positive saving or a positive cash down payment, and in either case $c_0^{\text{sal}} = w - s - \theta_{\text{cash}} p_0 h < w$. This proves the first sentence of (ii).

Next we show that $c_0^{\text{sal}} < c_0^R$ at every $w > \hat{w}$. If the renter is at or below her saving threshold, $c_0^R = w$ by Lemma 2(i), and we have just shown $c_0^{\text{sal}} < w$. If she is above it, Lemma 5, which was stated for either owner's problem, gives c_0^{sal} by (8) with the SAL buyer's own effective rent and shadow savings rate, and the comparison of second terms in the proof of (i) applies word for word, giving $c_0^{\text{sal}} < c_0^R$.

We now show that $\hat{w}$ is at most the renter's saving threshold. Suppose towards a con-

tradiction that $\hat{w}$ is strictly above the threshold. At $w = \hat{w}$, the SAL buyer consumes her entire endowment, $c_0^{\text{sal}} = \hat{w}$, while the renter, whose wealth exceeds her threshold, saves a positive amount by Lemma 2(i), so $c_0^R < \hat{w}$; hence $c_0^{\text{sal}} - c_0^R > 0$ at $\hat{w}$. At every $w > \hat{w}$, $c_0^{\text{sal}} - c_0^R < 0$ by the previous paragraph. But both consumption policies are continuous in w by Lemma 1(a), so their difference is continuous, and its limit as w falls to $\hat{w}$ from above is at most zero, which contradicts its value of strictly more than zero at $\hat{w}$. So $\hat{w}$ is at most the threshold.

Now we show the constancy of $v_R - v_O^{\text{sal}}$ on $(0, \hat{w}]$. At any $w \leq \hat{w}$, the SAL buyer's optimum is the wealth-free allocation, so $v_O^{\text{sal}}(w) - (1-\beta)\ln w$ is the maximized value of the wealth-free problem, which does not depend on w. Also, $w \leq \hat{w}$ is at most the renter's saving threshold, so by Lemma 2(i) the renter consumes her entire endowment and chooses the housing $\underline{h} + \beta(1+\rho)(y - q\underline{h})/\big(q(1+\beta\rho)\big)$, which does not depend on w; so $v_R(w) - (1-\beta)\ln w$ is the value of $\beta\ln(h-\underline{h}) + (1-\beta)\ln(y-qh)/(1+\rho)$ at that housing, which does not depend on w either. Subtracting, $v_R(w) - v_O^{\text{sal}}(w)$ is the difference of two numbers that do not depend on w, so it is constant on $(0, \hat{w}]$.

Finally, on $[\hat{w}, \infty)$: by Lemma 1(b), $v_R - v_O^{\text{sal}}$ is continuous and differentiable with derivative $(1-\beta)\left[1/c_0^R - 1/c_0^{\text{sal}}\right]$, and equals the integral of its derivative. At every $w > \hat{w}$ we have shown $c_0^{\text{sal}} < c_0^R$, so the derivative is strictly negative there, and, exactly as in the proof of (i), the integral between any two wealths $\hat{w} \leq w_1 < w_2$ is strictly negative. So $v_R - v_O^{\text{sal}}$ is strictly decreasing on $[\hat{w}, \infty)$. Continuity on all of $(0, \infty)$ is Lemma 1(b). This proves (ii).

Now we show (iii). Every allocation that is feasible in the unassisted problem at wealth w — a period-0 consumption, a house, and a cash share $\theta_{\text{cash}} \in [\underline{\theta}, 1]$ — is feasible in the SAL problem at the same wealth with $\theta_{\text{sal}} = 0$, and delivers the same lifetime expected utility there, because with $\theta_{\text{sal}} = 0$ the SAL repayment is zero and the two budget constraints coincide. So the maximum of the SAL problem is at least the maximum of the unassisted problem, $v_O^{\text{sal}}(w) \geq v_O(w)$, and subtracting both sides from $v_R(w)$ gives (iii).

Finally we show (iv). Ties are broken toward buying, and between the two ways of buying, toward the SAL. An agent with zero hassle cost compares $V_R = v_R$, $V_O = v_O + \phi$ (when unassisted ownership is feasible, that is, when $w > \underline{\theta} p_0 \underline{h}$), and $V_O^{\text{sal}} = v_O^{\text{sal}} + \phi$. Since $v_O^{\text{sal}} + \phi \geq v_O + \phi$ by (iii), buying with the SAL is her best alternative exactly when $v_O^{\text{sal}} + \phi \geq v_R$, that is, when $\phi \geq v_R(w) - v_O^{\text{sal}}(w)$. Without the program, she would buy exactly when unassisted ownership is feasible and it is at least as good as renting, that is, when $w > \underline{\theta} p_0 \underline{h}$ and $\phi \geq v_R(w) - v_O(w)$. So an agent who buys with the SAL is a marginal buyer — a buyer with the program who would rent without it — exactly when at least one of these two conditions fails, and an inframarginal taker — a buyer with or without the program — exactly when both hold. An agent with zero hassle cost who does not buy with the SAL, $\phi < v_R(w) - v_O^{\text{sal}}(w)$, also does not buy without the program, because by (iii) $\phi < v_R(w) - v_O^{\text{sal}}(w) \leq v_R(w) - v_O(w)$; she rents in either case, and is a renter never-taker. An agent with infinite hassle cost has $V_O^{\text{sal}} = -\infty$, so she compares only v_R and $v_O + \phi$, and buys unassisted exactly when $w > \underline{\theta} p_0 \underline{h}$ and $\phi \geq v_R(w) - v_O(w)$, with or without the program; she is a buyer never-taker if she buys and a renter never-taker if she rents.

For the last sentence of (iv), compare the set of owners with and without the program. Without the program, the owners are the agents with $w > \underline{\theta} p_0 \underline{h}$ and $\phi \geq v_R(w) - v_O(w)$, whatever their hassle cost. With the program, the owners are the agents with zero hassle cost and $\phi \geq v_R(w) - v_O^{\text{sal}}(w)$, together with the agents with infinite hassle cost, $w > \underline{\theta} p_0 \underline{h}$,

and $\phi \geq v_R(w) - v_O(w)$. An agent who owns with the program but not without it has zero hassle cost, $\phi \geq v_R(w) - v_O^{\text{sal}}(w)$, and either $w \leq \underline{\theta} p_0 \underline{h}$ or $\phi < v_R(w) - v_O(w)$: she is a marginal buyer. An agent who owns without the program but not with it would have to have $w > \underline{\theta} p_0 \underline{h}$, $\phi \geq v_R(w) - v_O(w)$, and $\phi < v_R(w) - v_O^{\text{sal}}(w)$, which is impossible by (iii). So the program changes the tenure of exactly the marginal buyers, from renting to owning, and the change in the homeownership rate is the population share of marginal buyers. This is what we wanted to show. □

Proposition 3 (Housing of marginal buyers)**.** *Suppose A1 and A2 hold, $\gamma < (p_H - p_L)/(p_H - p_0)$, and $\gamma \leq \gamma_{fair}$.*

(i) No buyer who uses the SAL makes a cash down payment, at any wealth. She consumes her entire endowment in period 0 when $w \leq \hat{w}$ and saves a positive amount when $w > \hat{w}$, with $\hat{w}$ as in Proposition 2.

(ii) At every wealth, an agent who buys with the SAL consumes strictly less housing than she would as a renter. In particular, every marginal buyer sacrifices housing when the SAL moves her from renting to owning.

Proof. First we show (i). Take any SAL buyer at her optimum, and suppose towards a contradiction that she makes a cash down payment, $\theta_{\text{cash}} > 0$. Consider the following marginal change: lower the cash down payment by a dollar and raise the SAL principal by a dollar, keeping the mortgage and the house fixed. This is feasible, because the cash share is strictly positive and can be lowered, and the combined share $\theta_{\text{cash}} + \theta_{\text{sal}}$ does not move. The change frees a period-0 dollar, which she uses in whichever way she values more: consuming it, worth $(1-\beta)/c_0$, or saving it, worth $(1-\beta)E[1/c_1]$. By the conventions discussion at the start of this appendix, at her optimum $(1-\beta)/c_0 \geq (1-\beta)E[1/c_1]$, so the freed dollar is worth at least $(1-\beta)E[1/c_1]$. The added dollar of SAL principal raises the SAL repayment by $1 + \gamma g^+$ dollars in state H — the principal plus the share γ of the appreciation $(p_H - p_0)/p_0$ per dollar of principal — and by 1 dollar in state L, and each period-1 dollar in state j is worth $(1-\beta)/c_1^j$ discounted by $1/(1+\rho)$; so the added principal costs $(1-\beta)\left[\pi(1+\gamma g^+)/c_1^H + (1-\pi)/c_1^L\right]/(1+\rho)$. So the derivative of lifetime expected utility along this change is at least

$$
\begin{aligned}
&(1-\beta)E\left[\frac{1}{c_1}\right] - \frac{1-\beta}{1+\rho}\left[\frac{\pi(1+\gamma g^+)}{c_1^H} + \frac{1-\pi}{c_1^L}\right] \\
&\quad = \frac{1-\beta}{1+\rho}\left\{(1+\rho)\left[\frac{\pi}{c_1^H} + \frac{1-\pi}{c_1^L}\right] - \frac{\pi(1+\gamma g^+)}{c_1^H} - \frac{1-\pi}{c_1^L}\right\} \\
&\quad = \frac{1-\beta}{1+\rho}\left\{\frac{\rho\pi}{c_1^H} + \frac{\rho(1-\pi)}{c_1^L} - \frac{\pi\gamma g^+}{c_1^H}\right\} \\
&\quad = \frac{1-\beta}{1+\rho}\left\{\rho\, E\left[\frac{1}{c_1}\right] - \frac{\pi\gamma g^+}{c_1^H}\right\} \\
&\quad = \frac{1-\beta}{1+\rho}\left\{\left(\rho - \pi\gamma g^+\right)E\left[\frac{1}{c_1}\right] + \pi\gamma g^+\left(E\left[\frac{1}{c_1}\right] - \frac{1}{c_1^H}\right)\right\} \\
&\quad = \frac{1-\beta}{1+\rho}\left\{\left(\rho - \pi\gamma g^+\right)E\left[\frac{1}{c_1}\right] + \pi(1-\pi)\gamma g^+\left(\frac{1}{c_1^L} - \frac{1}{c_1^H}\right)\right\},
\end{aligned}
$$

where the first equality puts everything over $1+\rho$ and writes out $E[1/c_1]$, the second collects the terms in $1/c_1^H$ and $1/c_1^L$ and the third recognizes $E[1/c_1]$, the fourth adds and subtracts $\pi\gamma g^+ E[1/c_1]$, and the fifth uses $E[1/c_1] - 1/c_1^H = \pi/c_1^H + (1-\pi)/c_1^L - 1/c_1^H = (1-\pi)\left(1/c_1^L - 1/c_1^H\right)$. Now we sign the final brace. Its first term is nonnegative exactly when $\rho \geq \pi\gamma g^+$, that is, when $\gamma \leq \rho/(\pi g^+) = \gamma_{\text{fair}}$, which holds by hypothesis; this term is the SAL's expected-cost advantage over cash. Its second term is strictly positive when $\gamma > 0$, because $c_1^L < c_1^H$ from the beginning of this appendix gives $1/c_1^L - 1/c_1^H > 0$; this term is the SAL's risk-sharing advantage. When $\gamma = 0$ the second term is zero and the brace is $\rho E[1/c_1] > 0$. So in every case the brace is strictly positive, and the change strictly raises her lifetime expected utility, which contradicts her optimality. So she makes no cash down payment. We note for use in Proposition 4 that the same argument applies word for word in the SAL problem without the period-0 constraint, where the freed dollar is worth exactly its saving value $(1-\beta)E[1/c_1]$.

Next we pin down $\hat{w}$. The computation just made, evaluated at the wealth-free allocation in the proof of Proposition 2(ii) (where $c_1^L < c_1^H$ holds as well), shows that

$$E\left[\frac{1}{c_1}\right] > \frac{1}{1+\rho}\left[\frac{\pi(1+\gamma g^+)}{c_1^H} + \frac{1-\pi}{c_1^L}\right]$$

at the wealth-free solution. So the larger of the two numbers in the definition of $\hat{w}$ is $E[1/c_1]$, and $\hat{w} = 1/E[1/c_1]$ at the wealth-free solution. Now take a SAL buyer at wealth w. If $w \leq \hat{w}$, her optimum is the wealth-free allocation (Proposition 2(ii)), so she consumes her entire endowment and saves nothing. If $w > \hat{w}$, her optimum has either positive saving or a positive cash down payment (proof of Proposition 2(ii)), and we have just shown that she makes no cash down payment, so she saves a positive amount. This proves (i).

Now we show (ii). We work casewise, according to whether the SAL buyer saves.

- *Case 1: $w \leq \hat{w}$.* By (i) the SAL buyer consumes her entire endowment and makes no cash down payment, so her saving is zero and her period-1 consumption in state j is $c_1^j = y - \omega_j h$; a unit of housing costs her nothing in period 0 and ω_j dollars of period-1 consumption in state j. The renter is also at or below her own saving threshold, because $\hat{w}$ is at most that threshold (proof of Proposition 2(ii)), so by Lemma 2(i) she consumes her entire endowment and $c_1 = y - qh$ for her; a unit of housing costs her q dollars of period-1 consumption. Each period-1 dollar in state j is worth $(1-\beta)/c_1^j$ discounted by $1/(1+\rho)$, and housing is unconstrained from above, so the derivative of lifetime expected utility with respect to h, holding the financing plan fixed, is zero at each agent's optimum:

$$\frac{\beta}{h-\underline{h}} = \frac{1-\beta}{1+\rho}\,E\left[\frac{\omega_j}{y-\omega_j h}\right] \quad \text{(SAL buyer)} \qquad \text{and} \qquad \frac{\beta}{h-\underline{h}} = \frac{1-\beta}{1+\rho}\cdot\frac{q}{y-qh} \quad \text{(renter)}.$$

  In each equation the left side, the marginal benefit of housing, has derivative $-\beta/(h-\underline{h})^2 < 0$ in h, so it is strictly decreasing. The right side, the marginal cost of housing, is strictly increasing: for the renter, $q/(y-qh)$ has derivative $q^2/(y-qh)^2 > 0$ in h; for the SAL buyer, $\omega_j/(y-\omega_j h)$ has derivative $\omega_j^2/(y-\omega_j h)^2 \geq 0$ in each state, and strictly positive in state L because $\omega_L > 0$, so the expectation is strictly increasing. A strictly

decreasing function and a strictly increasing function are equal at most once, and each agent's optimum exists by Lemma 1, so each equation has exactly one solution, which is that agent's housing.

We now compare the two marginal costs of housing at the same house. Consider the function $x \mapsto x/(y - xh)$ on $x < y/h$, at a fixed house h. Its first derivative is $[(y - xh) + xh]/(y - xh)^2 = y/(y - xh)^2 > 0$, and its second derivative is $2yh/(y - xh)^3 > 0$, so it is strictly increasing and strictly convex. Take any house the SAL buyer can afford in period 1, that is, one with $y - \omega_L h > 0$, so that $\omega_L < y/h$; then $\omega_H < \omega_L < y/h$ as well, and the renter can afford it too, because $q \leq \pi\omega_H + (1-\pi)\omega_L \leq \omega_L < y/h$ by A1 and the first fact at the beginning of this appendix. At any such house,

$$E\left[\frac{\omega_j}{y - \omega_j h}\right] = \pi \frac{\omega_H}{y - \omega_H h} + (1-\pi)\frac{\omega_L}{y - \omega_L h} > \frac{\pi\omega_H + (1-\pi)\omega_L}{y - [\pi\omega_H + (1-\pi)\omega_L]\, h} \geq \frac{q}{y - qh},$$

where the first inequality is strict convexity of $x \mapsto x/(y - xh)$ together with $\omega_H \neq \omega_L$ and $0 < \pi < 1$, and the second is A1, $q \leq \pi\omega_H + (1-\pi)\omega_L$, together with the fact that $x \mapsto x/(y - xh)$ is increasing. So the SAL buyer's marginal cost of housing strictly exceeds the renter's at every house the SAL buyer can afford.

Now take the renter's house. If the SAL buyer cannot afford it in period 1, then her own house, which she can afford, is smaller, and we are done. If she can afford it, then at the renter's house her marginal cost of housing strictly exceeds the renter's marginal cost, which equals the marginal benefit $\beta/(h - \underline{h})$ there, because the renter's first-order condition holds at the renter's house. So at the renter's house the SAL buyer's marginal benefit of housing falls strictly short of her marginal cost. Her marginal benefit is strictly decreasing and her marginal cost strictly increasing in h, so at every larger house her marginal benefit also falls short of her marginal cost, and her first-order condition can only be met at a strictly smaller house. So the SAL buyer's house is strictly smaller than the renter's.

- *Case 2:* $w > \hat{w}$. The SAL buyer saves a positive amount by (i), so $\hat{\rho} = \rho$ by the conventions discussion, and Lemma 5 gives her housing as the saving renter's formula (6) with the rent q replaced by her effective rent $u > q$:

$$h = \underline{h} + \frac{\beta(1+\rho)\left[(1+\rho)w + y - u\underline{h}\right]}{u\,(2+\rho-\beta)} = \underline{h} + \frac{\beta(1+\rho)}{2+\rho-\beta}\left[\frac{(1+\rho)w + y}{u} - \underline{h}\right].$$

The bracket is strictly decreasing in the rent in the denominator, because $(1+\rho)w + y > 0$, and strictly increasing in w. There are two sub-cases for the renter. If the renter saves, her housing is the same expression with q in place of u, by (6); since $q < u$, her bracket is strictly larger, and so is her house. If the renter does not save, then w is at most her saving threshold, and by Lemma 2(iii) her housing equals the same expression with q in place of u and with her saving threshold in place of w; since the threshold is at least w and the bracket is increasing in w, her house is at least the expression with q in place of u evaluated at w, which again is strictly larger than the SAL buyer's. In either sub-case the SAL buyer's housing is strictly smaller.

These cases are collectively exhaustive, which proves (ii). □

Proposition 4 (housing of inframarginal takers who are not strained)**.** *Suppose A1 and A2 hold and $\gamma < (p_H - p_L)/(p_H - p_0)$. Consider an agent who buys her home with or without the SAL and uses the SAL when it is available, and suppose the period-0 constraint carries no shadow value at her unassisted optimum — equivalently, by Lemma 4, her unassisted down-payment share is the cost-minimizing share θ^*_{cash}.*

(i) If the period-0 constraint carries no shadow value at her optimum with the SAL either, she consumes strictly more housing with the SAL than without it.

(ii) If $\gamma \leq \gamma_{fair}$, she consumes strictly more housing with the SAL than without it at every wealth, whether or not the condition in (i) holds.

Both parts hold for every $\beta \in (0, 1)$.

Proof. Throughout, by a problem *without the period-0 constraint* we mean the same problem with the constraint $c_0 + \theta_{\text{cash}} p_0 h \leq w$ deleted, so that saving $s = w - \theta_{\text{cash}} p_0 h - c_0$ may be negative; by Lemma 1(a) each such problem has a unique solution, continuous in wealth. We never characterize this problem for its own sake. We use it because of the following fact, which we now establish: an optimum of the original problem at which the period-0 constraint carries no shadow value is the unique optimum of the problem without the constraint. Recall from Lemma 4 that the constraint carries no shadow value exactly when a dollar of period-0 consumption is worth the same as a dollar saved, $1/c_0 = E[1/c_1]$. Take an optimum of the original problem at which this holds. Every marginal change in h, θ_{cash}, or θ_{sal} that is feasible in the problem without the constraint is also feasible in the original problem, because the only constraint deleted is the one on saving, and the derivative of lifetime expected utility along each such change is the same expression in both problems, because in both a period-0 dollar is worth $(1 - \beta)/c_0$, by the conventions discussion at the start of this appendix; so none of these changes raises utility. The one marginal change that the problem without the constraint allows and the original problem may not is to lower saving below zero, that is, when her saving is zero, to raise c_0 by a dollar at fixed h and financing plan; this gains $(1 - \beta)/c_0$ and loses $(1 - \beta)E[1/c_1]$, so the derivative of lifetime expected utility along it is $(1 - \beta)\,[1/c_0 - E[1/c_1]] = 0$. So no feasible marginal change of the problem without the constraint raises utility, and by Lemma 1 (sufficiency of the first-order conditions) the allocation is that problem's unique optimum. We also note that in any problem without the period-0 constraint, saving may be raised or lowered, so the derivative of lifetime expected utility with respect to saving, $(1-\beta)\,[E[1/c_1] - 1/c_0]$, is zero at the optimum: $1/c_0 = E[1/c_1]$ always holds there, and the shadow savings rate is $\hat{\rho} = \rho$. We prove four claims and then return to the main proof.

Claim 1: Under the hypothesis of the proposition, the agent's unassisted optimum is the optimum of the unassisted problem without the period-0 constraint. Moreover, the latter has down-payment share θ^*_{cash} at every wealth.

Proof of claim 1: The first statement is the fact just established, applied to the unassisted problem. For the second, we repeat the proof of Lemma 4 in the unassisted problem without the constraint. Consider raising the down-payment share by $d\theta$ at fixed h. This takes $p_0 h\, d\theta$ dollars out of saving, each worth $(1 - \beta)E[1/c_1] = (1 - \beta)/c_0$, and reduces the period-1

mortgage repayment in both states by $p_0 h\, d\theta$ times $1 + r(1-\theta_{\text{cash}}) + r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})$, each period-1 dollar being worth $(1-\beta)E[1/c_1]/(1+\rho)$. So the derivative of lifetime expected utility with respect to θ_{cash}, holding h fixed, is $(1-\beta)p_0 h$ times

$$\frac{1 + r(1-\theta_{\text{cash}}) + r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})}{1+\rho}\, E\left[\frac{1}{c_1}\right] - E\left[\frac{1}{c_1}\right]$$
$$= \frac{E[1/c_1]}{1+\rho}\Big\{\, [r(1-\theta_{\text{cash}}) + r'(1-\theta_{\text{cash}})(1-\theta_{\text{cash}})] - \rho \Big\},$$

which is the display in the proof of Lemma 4 with $\hat{\rho} = \rho$. It is signed by the difference between the marginal cost of mortgage financing at her share and ρ. Recall from Lemma 3 that the marginal cost of mortgage financing is strictly decreasing in θ_{cash}, exceeds ρ at $\underline{\theta}$, is below ρ at 1, and equals ρ exactly at θ^*_{cash}. There are three possibilities for her share. If it is interior, the derivative must be zero, so the marginal cost of mortgage financing at her share equals ρ, and her share is θ^*_{cash}. If it is $\underline{\theta}$, the share cannot be lowered, so optimality requires only that the derivative is at most zero, that is, that the marginal cost of mortgage financing at $\underline{\theta}$ is at most ρ; but it exceeds ρ, which contradicts. If it is 1, the share cannot be raised, so optimality requires only that the derivative is at least zero, that is, that $r(0) \geq \rho$; but $r(0) < \rho$, which contradicts. So her share is θ^*_{cash} at every wealth. //

Claim 2: Whenever the period-0 constraint carries no shadow value at the SAL buyer's optimum — in particular, whenever she saves a positive amount — her optimum is the optimum of the SAL problem without the period-0 constraint.

Proof of claim 2: This is the fact established at the start of the proof, applied to the SAL problem; if she saves a positive amount, then $1/c_0 = E[1/c_1]$ by the conventions discussion, so the constraint carries no shadow value. //

Claim 3: In either problem without the period-0 constraint, with the financing plan held fixed, housing is strictly increasing in wealth.

Proof of claim 3: Fix the financing plan, so that the state user costs ω_H, ω_L are fixed numbers, and fix a house h. Let the agent choose only c_0. Her period-1 consumption in state j is $c_1^j = (1+\rho)(w - c_0) + y - \omega_j h$, which is strictly decreasing in c_0, so $E[1/c_1]$ is strictly increasing in c_0 while $1/c_0$ is strictly decreasing, and the first-order condition for saving, $1/c_0 = E[1/c_1]$, has exactly one solution, which we write as $c_0(h, w)$. Call lifetime expected utility at $(c_0(h,w), h)$, as a function of h at the given w, the *profiled objective*.

We first show that the profiled objective is strictly concave in h. Take two houses $h_1 \neq h_2$, with $c_0^1 = c_0(h_1, w)$ and $c_0^2 = c_0(h_2, w)$, and $t \in (0,1)$. The direction from (c_0^1, h_1) to (c_0^2, h_2) changes housing, so by the first case in the proof of Lemma 1(a), lifetime expected utility is strictly concave along it: its value at $\big(tc_0^1 + (1-t)c_0^2,\ th_1 + (1-t)h_2\big)$ strictly exceeds t times its value at (c_0^1, h_1) plus $1-t$ times its value at (c_0^2, h_2). The profiled objective at $th_1 + (1-t)h_2$ is the maximum over c_0 at that house, so it is at least the value at $\big(tc_0^1 + (1-t)c_0^2,\ th_1 + (1-t)h_2\big)$, and hence strictly exceeds t times the profiled objective at h_1 plus $1-t$ times the profiled objective at h_2. This is strict concavity.

We next compute the derivative of the profiled objective in h. Lifetime expected utility is a function of (c_0, h); the profiled objective is this function evaluated at $(c_0(h,w), h)$, so by the chain rule its derivative in h is the partial derivative in h plus the partial derivative in c_0 times $\partial c_0(h,w)/\partial h$. The partial derivative in c_0 is $(1-\beta)\,[1/c_0 - E[1/c_1]]$, which is zero

at $c_0(h,w)$ by construction. So the derivative of the profiled objective in h is the partial derivative of lifetime expected utility in h at $(c_0(h,w),h)$. At a fixed financing plan, a unit of housing costs $\theta_{\text{cash}}p_0$ dollars in period 0, each worth $(1-\beta)/c_0 = (1-\beta)E[1/c_1]$, and $\omega_j-(1+\rho)\theta_{\text{cash}}p_0$ dollars in period 1 in state j, each worth $(1-\beta)/c_1^j$ discounted by $1/(1+\rho)$; so, exactly as in the first display of the proof of Lemma 5 with $\hat{\rho}=\rho$, the derivative of the profiled objective in h is

$$\frac{\beta}{h-\underline{h}} - \frac{1-\beta}{1+\rho}\,E\left[\frac{\omega_j}{c_1^j}\right], \qquad \text{with } c_1^j = (1+\rho)\big(w-c_0(h,w)\big)+y-\omega_j h.$$

We now show that, at a fixed house, this derivative is strictly increasing in w. Differentiate the first-order condition for saving, $1/c_0 = \pi/c_1^H + (1-\pi)/c_1^L$, in w at fixed h. Since $\partial c_1^j/\partial w = (1+\rho)\,(1-\partial c_0/\partial w)$ in both states,

$$\begin{aligned}
-\frac{1}{c_0^2}\frac{\partial c_0}{\partial w} &= -\pi\,\frac{1}{(c_1^H)^2}\frac{\partial c_1^H}{\partial w} - (1-\pi)\,\frac{1}{(c_1^L)^2}\frac{\partial c_1^L}{\partial w} = -(1+\rho)\left(1-\frac{\partial c_0}{\partial w}\right)E\left[\frac{1}{c_1^2}\right] \\
\Rightarrow \frac{\partial c_0}{\partial w}\left[\frac{1}{c_0^2}+(1+\rho)E\left[\frac{1}{c_1^2}\right]\right] &= (1+\rho)E\left[\frac{1}{c_1^2}\right] \\
\Rightarrow \frac{\partial c_0}{\partial w} &= \frac{(1+\rho)E\left[1/c_1^2\right]}{1/c_0^2+(1+\rho)E\left[1/c_1^2\right]} \in (0,1).
\end{aligned}$$

So $1-\partial c_0/\partial w > 0$, and c_1^j rises by the same strictly positive amount, $(1+\rho)(1-\partial c_0/\partial w)$, in both states. Then

$$\frac{\partial}{\partial w}E\left[\frac{\omega_j}{c_1^j}\right] = -\pi\,\frac{\omega_H}{(c_1^H)^2}\frac{\partial c_1^H}{\partial w} - (1-\pi)\,\frac{\omega_L}{(c_1^L)^2}\frac{\partial c_1^L}{\partial w} = -(1+\rho)\left(1-\frac{\partial c_0}{\partial w}\right)E\left[\frac{\omega_j}{(c_1^j)^2}\right].$$

We claim $E\left[\omega_j/(c_1^j)^2\right] > 0$. Its state-$L$ term, $(1-\pi)\omega_L/(c_1^L)^2$, is strictly positive because $\omega_L > 0$ from the beginning of this appendix. If $\omega_H \geq 0$, the state-H term is nonnegative and the claim follows. If $\omega_H < 0$, then, because $c_1^L < c_1^H$ gives $1/(c_1^L)^2 > 1/(c_1^H)^2$ and $\omega_L > 0$, replacing the factor $1/(c_1^L)^2$ in the state-L term by the smaller $1/(c_1^H)^2$ lowers it:

$$E\left[\frac{\omega_j}{(c_1^j)^2}\right] = \frac{\pi\omega_H}{(c_1^H)^2} + \frac{(1-\pi)\omega_L}{(c_1^L)^2} > \frac{\pi\omega_H}{(c_1^H)^2} + \frac{(1-\pi)\omega_L}{(c_1^H)^2} = \frac{\pi\omega_H+(1-\pi)\omega_L}{(c_1^H)^2} \geq \frac{q}{(c_1^H)^2} > 0,$$

where the second inequality is A1. So $\partial E[\omega_j/c_1^j]/\partial w < 0$, and the derivative of the profiled objective in h, which is $\beta/(h-\underline{h})$ minus a positive multiple of $E[\omega_j/c_1^j]$, is strictly increasing in w at every fixed house.

Finally, let $w_2 > w_1$ and let h_1 be the optimal house at w_1; it is interior, so the derivative of the profiled objective at w_1 is zero at h_1. By the previous paragraph, the derivative of the profiled objective at w_2 is strictly positive at h_1. Let h_2 be the optimal house at w_2, where the derivative of the profiled objective at w_2 is zero. The profiled objective at w_2 is strictly concave, so its derivative is strictly decreasing in h; since that derivative is strictly positive at h_1 and zero at h_2, we must have $h_2 > h_1$. //

Claim 4: Fix a wealth w at which the optimum of the SAL problem without the period-

0 constraint uses the SAL, and call its SAL share $\tau^* > 0$. Then housing in the SAL problem without the constraint strictly exceeds housing in the unassisted problem without the constraint at w.

Proof of claim 4: The proof has four steps. First we define a family of problems indexed by a cap τ on the SAL share, running from the unassisted problem at $\tau = 0$ to the SAL problem at $\tau = \tau^*$, and show that the cap binds throughout. Second we pin down the rest of the financing plan at each cap, and compute how the state user costs move with the cap. Third we show that, at each cap, the SAL buyer does not want to lower her SAL share. Fourth we differentiate the first-order conditions for housing and saving in the cap and show that housing rises with it.

Step 1. For each $\tau \in [0, \tau^*]$, consider the SAL problem without the period-0 constraint and with the additional constraint $\theta_{\text{sal}} \leq \tau$; call it the *capped problem* at τ, and its unique solution (Lemma 1(a)) the *capped optimum*. At $\tau = 0$, the cap together with $\theta_{\text{sal}} \geq 0$ forces $\theta_{\text{sal}} = 0$, so the capped problem is the unassisted problem without the constraint, by the argument in the proof of Proposition 2(iii). At $\tau = \tau^*$, the optimum of the SAL problem without the constraint satisfies the cap, so it is feasible in the capped problem; the capped problem has a smaller feasible set than the SAL problem without the constraint, so its maximum is at most the latter's, and the allocation attains that maximum; so it is the capped optimum. For $0 < \tau < \tau^*$, suppose towards a contradiction that the cap does not bind at the capped optimum, $\theta_{\text{sal}} < \tau$. Then every marginal change that is feasible in the SAL problem without the constraint is feasible in the capped problem, because the only constraint that differs is slack; so no feasible marginal change of the SAL problem without the constraint raises utility at the capped optimum, and by Lemma 1 (sufficiency of the first-order conditions) the capped optimum is the unique optimum of the SAL problem without the constraint. But that optimum has SAL share $\tau^* > \tau > \theta_{\text{sal}}$, which contradicts. So the SAL share at the capped optimum equals τ for every $\tau \in [0, \tau^*]$. By Lemma 1(a) (an upper bound placed on the SAL share, with the period-0 constraint deleted), the capped optimum is continuous in τ.

Step 2. Given the SAL share τ, we pin down the cash share, by repeating the argument of claim 1 with the SAL share held at τ. Write $\theta = \theta_{\text{cash}} + \tau$ for the combined down-payment share. Consider raising the cash share by $d\theta_{\text{cash}}$ at fixed h and fixed SAL share, so that the mortgage share falls by the same amount. This takes $p_0 h\, d\theta_{\text{cash}}$ dollars out of saving, each worth $(1-\beta)E[1/c_1] = (1-\beta)/c_0$, and reduces the period-1 mortgage repayment $(1+r(1-\theta))(1-\theta)p_0 h$ in both states by $p_0 h\, d\theta_{\text{cash}}$ times $1 + r(1-\theta) + r'(1-\theta)(1-\theta)$, each period-1 dollar being worth $(1-\beta)E[1/c_1]/(1+\rho)$. So, as in the display in the proof of claim 1, the derivative of lifetime expected utility with respect to θ_{cash}, holding h and the SAL share fixed, is $(1-\beta)p_0 h\, E[1/c_1]/(1+\rho)$ times $[r(1-\theta) + r'(1-\theta)(1-\theta)] - \rho$, where the marginal cost of mortgage financing is now evaluated at the combined share θ. By Lemma 3, this difference is strictly positive for $\theta < \theta^*_{\text{cash}}$ and strictly negative for $\theta > \theta^*_{\text{cash}}$. There are two cases. If $\tau \leq \theta^*_{\text{cash}}$: a cash share with $\theta_{\text{cash}} + \tau < \theta^*_{\text{cash}}$ is not optimal, because the combined share is then below 1 and raising the cash share strictly raises utility; and a cash share with $\theta_{\text{cash}} + \tau > \theta^*_{\text{cash}}$ is not optimal either, because then $\theta_{\text{cash}} > \theta^*_{\text{cash}} - \tau \geq 0$, so the cash share can be lowered, and lowering it strictly raises utility. So the cash share is $\theta^*_{\text{cash}} - \tau \geq 0$, and the combined share is θ^*_{cash}. If $\tau \geq \theta^*_{\text{cash}}$: any positive cash share gives a combined share $\theta_{\text{cash}} + \tau > \theta^*_{\text{cash}}$, so lowering it strictly raises utility, and the cash share is

zero; the combined share is τ and the mortgage share is $1-\tau$. (Both combined shares lie in $[\underline{\theta}, 1]$, since $\underline{\theta} < \theta^*_{\text{cash}} < 1$ by Lemma 3 and $\tau \leq \tau^* \leq 1$.) We call $\tau \leq \theta^*_{\text{cash}}$ the *first stretch* and $\tau \geq \theta^*_{\text{cash}}$ the *second stretch*; if $\tau^* \leq \theta^*_{\text{cash}}$, only the first stretch arises, and the argument below covers both possibilities.

Along the first stretch, each unit of SAL share replaces a unit of cash share at a fixed mortgage share $1-\theta^*_{\text{cash}}$, and, substituting $\theta_{\text{cash}} = \theta^*_{\text{cash}} - \tau$ and $\theta_{\text{sal}} = \tau$ into the definition of the state user costs at the beginning of this appendix,

$$\omega_j = (1+\rho)(\theta^*_{\text{cash}} - \tau)p_0 + (1 + r(1-\theta^*_{\text{cash}}))(1-\theta^*_{\text{cash}})p_0 + \tau\left[p_0 + \gamma(p_j - p_0)^+\right] - p_j.$$

Only the first and third terms involve τ. Differentiating in τ, and using $(p_L - p_0)^+ = 0$ and $(p_H - p_0)^+ = p_0 g^+$,

$$\frac{d\omega_L}{d\tau} = -(1+\rho)p_0 + p_0 = -\rho p_0 < 0, \qquad \frac{d\omega_H}{d\tau} = -(1+\rho)p_0 + p_0 + \gamma p_0 g^+ = p_0\left(\gamma g^+ - \rho\right).$$

Along the second stretch, each unit of SAL share replaces a unit of mortgage share at a cash share of zero, and, substituting $\theta_{\text{cash}} = 0$ and $\theta_{\text{sal}} = \tau$,

$$\omega_j = (1 + r(1-\tau))(1-\tau)p_0 + \tau\left[p_0 + \gamma(p_j - p_0)^+\right] - p_j.$$

Differentiating the first term in τ by the product rule, $\frac{d}{d\tau}\left[(1 + r(1-\tau))(1-\tau)p_0\right] = -r'(1-\tau)(1-\tau)p_0 - (1 + r(1-\tau))p_0 = -p_0\left[1 + r(1-\tau) + r'(1-\tau)(1-\tau)\right]$, so

$$\frac{d\omega_L}{d\tau} = -p_0\left[1 + r(1-\tau) + r'(1-\tau)(1-\tau)\right] + p_0 = -\left[r(1-\tau) + r'(1-\tau)(1-\tau)\right]p_0 < 0,$$
$$\frac{d\omega_H}{d\tau} = \frac{d\omega_L}{d\tau} + \gamma(p_H - p_0),$$

where the inequality uses $r \geq 0$, $r' > 0$, and $1-\tau \geq 0$. On both stretches, then, $d\omega_L/d\tau < 0$, while $d\omega_H/d\tau$ may take either sign.

Step 3. We show that at any cap τ strictly inside a stretch, the capped optimum satisfies

$$\pi \frac{d\omega_H/d\tau}{c_1^H} + (1-\pi)\frac{d\omega_L/d\tau}{c_1^L} \leq 0. \tag{10}$$

Suppose towards a contradiction that the left side is strictly positive. Consider the following marginal change at the capped optimum: lower the SAL share by $d\tau > 0$ and replace it with the instrument it displaces on that stretch — cash on the first stretch, mortgage on the second — at fixed h and c_0. This is feasible: the SAL share τ is strictly positive and may be lowered under the cap, which is an upper bound; on the first stretch, raising the cash share is always feasible; on the second stretch, raising the mortgage share is feasible because $\tau > \theta^*_{\text{cash}} > \underline{\theta}$ leaves the combined share above $\underline{\theta}$; and with c_0 fixed, any period-0 dollars needed come out of saving, which is unrestricted in sign. The financing plan after the change is the plan on the same stretch at the SAL share $\tau - d\tau$, so the state user costs change by $-(d\omega_j/d\tau)\,d\tau$. Since $c_1^j = (1+\rho)(w - c_0) + y - \omega_j h$ with w, c_0, and h fixed, period-1 consumption in state j changes by $+h\,(d\omega_j/d\tau)\,d\tau$. Each period-1 dollar in state j

is worth $(1-\beta)/c_1^j$ discounted by $1/(1+\rho)$, so the change in lifetime expected utility is

$$\frac{1-\beta}{1+\rho}\, h\, d\tau \left[\pi\, \frac{d\omega_H/d\tau}{c_1^H} + (1-\pi)\, \frac{d\omega_L/d\tau}{c_1^L}\right],$$

which is strictly positive. So the change raises utility, which contradicts the optimality of the capped optimum. This proves (10).

Step 4. It remains to show that housing is strictly increasing in τ along each stretch. At each τ, with the financing plan fixed at the plan pinned down in step 2, the capped optimum solves a problem in (h, c_0) alone, with $c_1^j = (1+\rho)(w-c_0) + y - \omega_j h$, and the first-order conditions for housing and saving, both of which hold with equality because h is interior and saving is unrestricted in sign, are

$$\frac{\beta}{h-\underline{h}} = \frac{1-\beta}{1+\rho}\, E\left[\frac{\omega_j}{c_1^j}\right] \qquad \text{(housing)}, \qquad\qquad \frac{1}{c_0} = E\left[\frac{1}{c_1}\right] \qquad \text{(saving)},$$

where the housing condition is the derivative computed in the proof of claim 3 set to zero. As τ moves along a stretch, the plan moves with it and ω_H, ω_L become the functions of τ written in step 2, while h and c_0 at the capped optimum are the solution of this pair of equations. The matrix of partial derivatives of the two conditions in (h, c_0) is the matrix of second derivatives of lifetime expected utility in (h, c_0) at a fixed plan, which is negative definite, because by the first case in the proof of Lemma 1(a) lifetime expected utility is strictly concave along every direction that changes h or c_0; so by the implicit function theorem, h and c_0 at the capped optimum are differentiable functions of τ on the interior of each stretch. We differentiate both conditions in τ, writing a prime for $d/d\tau$. By the chain rule applied to $c_1^j = (1+\rho)(w-c_0) + y - \omega_j h$,

$$\frac{dc_1^j}{d\tau} = -(1+\rho)c_0' - \omega_j' h - \omega_j h'.$$

Differentiating the saving condition, $1/c_0 = \pi/c_1^H + (1-\pi)/c_1^L$, in τ:

$$\begin{aligned} -\frac{c_0'}{c_0^2} &= -\pi\, \frac{1}{(c_1^H)^2}\frac{dc_1^H}{d\tau} - (1-\pi)\, \frac{1}{(c_1^L)^2}\frac{dc_1^L}{d\tau} = E\left[\frac{(1+\rho)c_0' + \omega_j' h + \omega_j h'}{(c_1^j)^2}\right] \\ &= (1+\rho)\, c_0'\, E\left[\frac{1}{c_1^2}\right] + h\, E\left[\frac{\omega_j'}{(c_1^j)^2}\right] + h'\, E\left[\frac{\omega_j}{(c_1^j)^2}\right] \\ \Rightarrow\; -c_0'\left[\frac{1}{c_0^2} + (1+\rho)E\left[\frac{1}{c_1^2}\right]\right] &= h\, E\left[\frac{\omega_j'}{(c_1^j)^2}\right] + h'\, E\left[\frac{\omega_j}{(c_1^j)^2}\right], \end{aligned}$$

where the last line moves the term in c_0' to the left. Differentiating the housing condition in τ, by the quotient rule applied to ω_j/c_1^j:

$$-\frac{\beta\, h'}{(h-\underline{h})^2} = \frac{1-\beta}{1+\rho}\, E\left[\frac{\omega_j'}{c_1^j} - \frac{\omega_j}{(c_1^j)^2}\frac{dc_1^j}{d\tau}\right] = \frac{1-\beta}{1+\rho}\, E\left[\frac{\omega_j'}{c_1^j} + \frac{\omega_j\left[(1+\rho)c_0' + \omega_j' h + \omega_j h'\right]}{(c_1^j)^2}\right].$$

Inside the expectation, collect the two terms in ω_j' over the common denominator $(c_1^j)^2$: $\omega_j'/c_1^j + h\,\omega_j\omega_j'/(c_1^j)^2 = \omega_j'\left[c_1^j + \omega_j h\right]/(c_1^j)^2$, and by the budget constraint $c_1^j + \omega_j h = (1+\rho)(w-c_0)+y$, which does not depend on the state. So

$$-\frac{\beta\,h'}{(h-\underline{h})^2} = \frac{1-\beta}{1+\rho}\left\{[(1+\rho)(w-c_0)+y]\,E\left[\frac{\omega_j'}{(c_1^j)^2}\right] + (1+\rho)\,c_0'\,E\left[\frac{\omega_j}{(c_1^j)^2}\right] + h'\,E\left[\frac{\omega_j^2}{(c_1^j)^2}\right]\right\}.$$

Now solve the differentiated saving condition for c_0',

$$c_0' = -\,\frac{h\,E\left[\omega_j'/(c_1^j)^2\right] + h'\,E\left[\omega_j/(c_1^j)^2\right]}{1/c_0^2 + (1+\rho)E\left[1/c_1^2\right]},$$

substitute it into the differentiated housing condition, and collect the terms in h' on the left and the terms in $E\left[\omega_j'/(c_1^j)^2\right]$ on the right:

$$h'\left\{-\frac{\beta}{(h-\underline{h})^2} - \frac{1-\beta}{1+\rho}\left[E\left[\frac{\omega_j^2}{(c_1^j)^2}\right] - \frac{(1+\rho)\,E\left[\omega_j/(c_1^j)^2\right]^2}{1/c_0^2+(1+\rho)E\left[1/c_1^2\right]}\right]\right\}$$
$$= \frac{1-\beta}{1+\rho}\,E\left[\frac{\omega_j'}{(c_1^j)^2}\right]\left\{(1+\rho)(w-c_0)+y - \frac{(1+\rho)\,h\,E\left[\omega_j/(c_1^j)^2\right]}{1/c_0^2+(1+\rho)E\left[1/c_1^2\right]}\right\}.$$

We sign the two braces. The brace on the left is strictly negative. Its first term, $-\beta/(h-\underline{h})^2$, is strictly negative, so it suffices that the bracket is nonnegative. By the Cauchy–Schwarz inequality applied to the pair $1/c_1^j$ and ω_j/c_1^j, $E\left[\omega_j/(c_1^j)^2\right]^2 = E\left[(1/c_1^j)(\omega_j/c_1^j)\right]^2 \le E\left[1/c_1^2\right]E\left[\omega_j^2/(c_1^j)^2\right]$; and $(1+\rho)/\left[1/c_0^2+(1+\rho)E[1/c_1^2]\right] < 1/E[1/c_1^2]$ because $1/c_0^2>0$. So the term subtracted in the bracket is at most $E\left[\omega_j/(c_1^j)^2\right]^2/E[1/c_1^2] \le E\left[\omega_j^2/(c_1^j)^2\right]$, and the bracket is nonnegative. The brace on the right is strictly positive. Putting it over the common denominator $1/c_0^2+(1+\rho)E[1/c_1^2]$, which is positive, its numerator is

$$[(1+\rho)(w-c_0)+y]\left[\frac{1}{c_0^2}+(1+\rho)E\left[\frac{1}{c_1^2}\right]\right] - (1+\rho)\,h\,E\left[\frac{\omega_j}{(c_1^j)^2}\right]$$
$$= \frac{(1+\rho)(w-c_0)+y}{c_0^2} + (1+\rho)\,E\left[\frac{(1+\rho)(w-c_0)+y-\omega_j h}{(c_1^j)^2}\right]$$
$$= \frac{(1+\rho)(w-c_0)+y}{c_0^2} + (1+\rho)\,E\left[\frac{1}{c_1}\right] > 0,$$

where the second equality uses $(1+\rho)(w-c_0)+y-\omega_j h = c_1^j$, and the inequality uses $(1+\rho)(w-c_0)+y = c_1^L + \omega_L h > 0$. So, dividing both sides of the collected equation by the strictly negative brace on the left, h' equals a strictly negative number times $E\left[\omega_j'/(c_1^j)^2\right]$, and h' has the sign opposite to $E\left[\omega_j'/(c_1^j)^2\right]$.

Finally we show $E\left[\omega_j'/(c_1^j)^2\right] < 0$. Its state-$L$ term, $(1-\pi)\omega_L'/(c_1^L)^2$, is strictly negative because $\omega_L' < 0$ on both stretches (step 2). If $\omega_H' \le 0$, the state-H term is at most zero and we are done. If $\omega_H' > 0$, write each term as $1/c_1^j$ times ω_j'/c_1^j; since $1/c_1^H < 1/c_1^L$ and

$\pi\omega'_H/c_1^H > 0$, replacing the factor $1/c_1^H$ in the state-H term by the larger $1/c_1^L$ raises it:

$$E\left[\frac{\omega'_j}{(c_1^j)^2}\right] = \frac{1}{c_1^H}\cdot\frac{\pi\omega'_H}{c_1^H} + \frac{1}{c_1^L}\cdot\frac{(1-\pi)\omega'_L}{c_1^L} < \frac{1}{c_1^L}\left[\pi\,\frac{\omega'_H}{c_1^H} + (1-\pi)\,\frac{\omega'_L}{c_1^L}\right] \le 0,$$

where the last inequality is (10). So $h' > 0$ on the interior of each stretch: housing at the capped optimum is strictly increasing in τ there. Housing is also continuous in τ on $[0, \tau^*]$ by step 1. A function that is continuous on a closed interval and strictly increasing on its interior is strictly increasing on the closed interval: for any $0 \le a < b \le \tau^*$ within one stretch, pick $a < a' < b' < b$, so that housing at a' is strictly below housing at b', and let $a' \to a$ and $b' \to b$ to get housing at a at most housing at a', strictly below housing at b', at most housing at b. Applying this on $[0, \tau^*]$ if $\tau^* \le \theta^*_{\text{cash}}$, and on $[0, \theta^*_{\text{cash}}]$ and then on $[\theta^*_{\text{cash}}, \tau^*]$ otherwise, housing at τ^* strictly exceeds housing at 0, since $\tau^* > 0$. Housing at τ^* is housing in the SAL problem without the constraint, and housing at 0 is housing in the unassisted problem without the constraint (step 1). Note that nothing in claims 1 through 4 uses γ beyond the maintained bound. //

Now we return to the main proof. First we show (i). Under the condition in (i), her optimum with the SAL is the optimum of the SAL problem without the period-0 constraint at w (claim 2). By hypothesis she uses the SAL, so that optimum has a strictly positive SAL share, and claim 4 applies at w: her housing with the SAL strictly exceeds housing in the unassisted problem without the constraint at w. By claim 1, the latter is her unassisted housing. This proves (i).

Now we show (ii). Suppose $\gamma \le \gamma_{\text{fair}}$. By Proposition 3(i), no SAL buyer makes a cash down payment; and the optimum of the SAL problem without the period-0 constraint makes no cash down payment either, because, as noted in the proof of Proposition 3(i), the swap argument applies there word for word. So the optimum of the SAL problem without the constraint has combined share $\theta_{\text{cash}} + \theta_{\text{sal}} = \theta_{\text{sal}} \ge \underline{\theta} > 0$ at every wealth, and claim 4 applies at every wealth. We work casewise, according to whether the SAL buyer saves.

- *Case 1:* $w > \hat{w}$. The SAL buyer saves a positive amount (Proposition 3(i)), so by claim 2 the condition in (i) holds, and (i) gives the result.

- *Case 2:* $w \le \hat{w}$. The SAL buyer consumes her entire endowment and makes no cash down payment (Proposition 3(i)), and her optimum is the wealth-free allocation of the proof of Proposition 2(ii), whose housing and SAL share are the same at every $w \le \hat{w}$. We claim that the allocation consisting of $c_0 = \hat{w}$, zero saving, no cash down payment, and the wealth-free housing and SAL share is the optimum of the SAL problem without the period-0 constraint at wealth $\hat{w}$. By Lemma 1 (sufficiency of the first-order conditions) it suffices to check that no feasible marginal change raises lifetime expected utility there; the feasible marginal changes are those listed in the proof of Proposition 2(ii), together with raising c_0, which the problem without the constraint allows. Changes in h and exchanges of SAL for mortgage do not raise utility, by the wealth-free problem's own first-order conditions. A cash dollar in place of a dollar of SAL principal does not raise utility, because at $c_0 = \hat{w}$ the period-0 dollar is worth $(1-\beta)/\hat{w} = (1-\beta)E[1/c_1]$ (recall from the proof of Proposition 3(i) that $\hat{w} = 1/E[1/c_1]$ at the wealth-free solution), and the swap computation in that

proof shows that a dollar of SAL principal costs strictly less than $(1-\beta)E[1/c_1]$; a cash dollar on top of the SAL is, as before, this change combined with an increase in the SAL share, neither of which raises utility. Finally, raising or lowering saving does not raise utility, because the derivative of lifetime expected utility with respect to saving, $(1-\beta)\left[E[1/c_1]-1/c_0\right]$, is zero at $c_0=\hat{w}$. So her housing with the SAL equals housing in the SAL problem without the constraint at wealth $\hat{w}$. Now chain the comparisons: her unassisted housing at w equals housing in the unassisted problem without the constraint at w (claim 1); this is at most housing in the unassisted problem without the constraint at $\hat{w}$, because $w \leq \hat{w}$ and the financing plan is θ^*_{cash} at both wealths (claims 1 and 3, with equality if $w=\hat{w}$); and this is strictly less than housing in the SAL problem without the constraint at $\hat{w}$ (claim 4), which is her housing with the SAL.

These cases are collectively exhaustive, which proves (ii). □

D Trinomial selection model

Results reported in the main text use a trinomial instrumental variables model following Angrist, Santos and Tecchio (2026) to estimate causal effects of Dream for All utilization relative to buying unassisted or renting. This section discusses an alternative approach following Kline and Walters (2016) for estimating the same causal effects, which parametrizes preferences and potential outcomes introduced in Section 6 to motivate a two-step control function estimator, akin to a two-dimensional Heckit. For conciseness, and also to map explicitly to the notation of Kline and Walters (2016), we label the three treatments h (buy with DFA), c (buy unassisted), and n (rent), and write $Z_i = \text{LOTTERY}_i$.

Unlike the main analysis sample, the sample for this section includes waitlisted applicants who subsequently received a Dream for All offer forfeited by a first-round recipient. These applicants received offers in two cohorts, before December 2024 and during 2025, and we let $\mathbf{C}_i$ denote the pair of cohort indicators, which are zero for applicants who were never offered a loan and for applicants offered a loan in the first round. Z_i here indicates any offer, whether in the first round or from the waitlist. The role of the waitlist is to identify selection into Dream for All homebuying, as explained below. The estimands remain the first-round-offer complier groups and subLATEs of Section 6, so potential choices $D_i(1)$ throughout refer to a first-round offer.

We introduce a choice model normalizing the value of renting to zero, and posit that applicants' payoffs over housing choices are described by

$$U_i(h, Z_i, \mathbf{C}_i) = \psi_h(W_i, Z_i, \mathbf{C}_i) + v_{ih}, \quad U_i(c, \mathbf{C}_i) = \psi_c(W_i, \mathbf{C}_i) + v_{ic}, \quad U_i(n) = 0$$

where W_i stacks vectors of baseline applicant characteristics X_i and region indicators $\mathbf{R}_i$. The offer cohort enters the payoffs to buying with and without Dream for All because a waitlisted applicant's offer arrived months after the first round, by which point some waitlisted applicants had already bought unassisted. For a first-round offer, $\mathbf{C}_i = 0$ and the offer is excluded from the payoff to buying unassisted. The utility shocks (v_{ih}, v_{ic}) are unobserved differences in demand for buying a home with and without Dream for All relative to renting. We assume the joint distribution of these shocks conditional on W_i is normal with the specified covariance structure

$$(v_{ih}, v_{ic}) \mid W_i, Z_i, \mathbf{C}_i \sim \mathcal{N}\left(0, \begin{bmatrix} 1 & \varrho(W_i) \\ \varrho(W_i) & 1 \end{bmatrix}\right)$$

We model potential outcomes' CEFs under choice $d \in \{h, c, n\}$ conditional on covariates and shocks as:

$$E\left[Y_i(d) \mid W_i, Z_i, \mathbf{C}_i, v_{ih}, v_{ic}\right] = \mu_d(W_i) + \kappa_{dh} v_{ih} + \kappa_{dc} v_{ic} \tag{11}$$

The $\{\kappa_{dh}, \kappa_{dc}\}$ coefficients in equation 11 describe the nature of selection on unobservables.[2]

[2] If $\kappa_{dh} = \kappa_h > 0$, then conditional on observables, selection into Dream for All homebuying is governed by potential outcome levels: the applicants most likely to use Dream for All assistance have higher values of the outcome under all housing choices. A scenario with $\kappa_{hh} > 0$ and $\kappa_{nh} = -\kappa_{hh}$ is usually characterized as Roy (1951)-style selection on gains: those most likely to use Dream for All assistance experience larger

Equation 11 embeds an exclusion restriction beyond that of Section 6: the offer cohort affects outcomes only through the tenure and financing choice and the taste shocks, not through $\mu_d(W_i)$. Kline and Walters (2016) show by iterated expectations that equation 11 implies

$$E\left[Y_i \mid W_i, Z_i, \mathbf{C}_i, D_i = d\right] = \mu_d\left(W_i\right) + \kappa_{dh}\lambda_h\left(W_i, Z_i, \mathbf{C}_i, d\right) + \kappa_{dc}\lambda_c\left(W_i, Z_i, \mathbf{C}_i, d\right), \tag{12}$$

where the selection correction terms $\lambda_h(W_i, Z_i, \mathbf{C}_i, D_i) \equiv E\left[v_{ih} \mid W_i, Z_i, \mathbf{C}_i, D_i\right]$ and $\lambda_c(W_i, Z_i, \mathbf{C}_i, D_i) \equiv E\left[v_{ic} \mid W_i, Z_i, \mathbf{C}_i, D_i\right]$ are evaluations of the joint normal cumulative distribution functions of the taste shocks $\{v_{ic}, v_{ih}\}$.

Equation 11 gives inframarginal takers' average effect of Dream for All utilization conditional on covariates as

$$\begin{aligned} E\left[Y_i(h) - Y_i(c) \mid W_i, D_i(1) = h, D_i(0) = c\right] = \mu_h(W_i) - \mu_c(W_i) \\ + \left(\kappa_{hh} - \kappa_{ch}\right) E\left[v_{ih} \mid W_i, D_i(1) = h, D_i(0) = c\right] \\ + \left(\kappa_{hc} - \kappa_{cc}\right) E\left[v_{ic} \mid W_i, D_i(1) = h, D_i(0) = c\right] \end{aligned}$$

and likewise for marginal buyers with c replaced by n. The conditional means of the taste shocks are averages over the region of the (v_{ih}, v_{ic}) distribution in which an applicant buys unassisted without a first-round offer and with Dream for All with one. Under monotonicity, this region is the part of the buy-unassisted region without an offer which lies outside the buy-unassisted region with an offer, so its conditional means are differences of the control functions in the two observed cells:

$$\begin{aligned} &E\left[v_{ih} \mid W_i, D_i(1) = h, D_i(0) = c\right] \\ &\quad = \frac{P\left(D_i = c \mid W_i, 0, 0\right)\lambda_h(W_i, 0, 0, c) - P\left(D_i = c \mid W_i, 1, 0\right)\lambda_h(W_i, 1, 0, c)}{P\left(D_i = c \mid W_i, 0, 0\right) - P\left(D_i = c \mid W_i, 1, 0\right)} \end{aligned}$$

where the choice probabilities are those of the choice model with $\mathbf{C}_i = 0$, and the expressions for v_{ic}, and for marginal buyers, are analogous. We average conditional subLATEs over Dream for All buyers, weighting each by the model's probability that an applicant with her covariates is an inframarginal taker rather than a marginal buyer, $P\left(D_i(0) = c \mid D_i(1) = h, W_i\right)$, or vice versa.

Identification of the selection model amounts to identification of the correlation parameter $\varrho(W_i)$ between taste shocks $\{v_{ic}, v_{ih}\}$ and of the selection coefficients $\{\kappa_{dh}, \kappa_{dc}\}, d \in \{h, c, n\}$. The taste correlation parameter is identified by the choice probabilities $P(D_i = d \mid W_i, Z_i = \ell, \mathbf{C}_i)$ for each alternative d. The exclusion of a first-round Dream for All offer from agents' utility under alternatives c and n implies that the offer generates variation in choice probabilities which is independent of observed characteristics, aiding identification.

Identification of the selection coefficients relies on variation in the control functions within each treatment arm which is not a function of the covariates. For the buy-unassisted and rent arms, the offer supplies this variation: applicants who rent or buy unassisted are observed both with and without an offer, and an offer changes the region of the taste shock distribution

increases in the outcome when they switch from renting to buying with Dream for All. The "selection on gains" language intended for outcomes like wages translates rather awkwardly to outcomes like housing consumption and monthly mortgage payments.

in which these choices are made, and hence λ_h and λ_c, holding W_i fixed. The main analysis sample contains no such variation for the Dream for All arm. Because non-compliance is one-sided, every Dream for All buyer received an offer, so $\lambda_h(W_i, 1, 0, h)$ and $\lambda_c(W_i, 1, 0, h)$ vary only through W_i, and κ_{hh} and κ_{hc} would be identified only by the nonlinearity of the control functions in the covariates. The waitlist supplies the missing variation. Because some waitlisted applicants bought unassisted while waiting, the share of offered applicants who buy unassisted differs across offer cohorts, which the choice model captures through the cohort indicators in ψ_h and ψ_c. The cohort thereby changes the region of the taste shock distribution in which offered applicants buy with Dream for All, so the control functions among Dream for All buyers vary across cohorts conditional on W_i, identifying κ_{hh} and κ_{hc} under the exclusion of the cohort from $\mu_h(W_i)$.

A further identification assumption is additive separability of potential outcomes in observables and unobservables: selection on unobservables must not vary across values of the covariates. To build intuition, suppose that applicants with three-person households are more likely than single applicants to buy homes without assistance when denied access to a Dream for All loan. The model allows Dream for All and buying a home without Dream for All to have different average treatment effects on the price of the home purchased by applicants with smaller or larger household sizes. However, it rules out the possibility that applicants with larger households sort into Dream for All on the basis of the more expensive homes it would let them buy, while applicants in smaller households exhibit no sorting on these gains.

We estimate the model in two steps. We approximate $\psi_h(W, Z, \mathbf{C})$ and $\psi_c(W, \mathbf{C})$ with linear interactions between Z and the covariates W, to which we add the cohort indicators $\mathbf{C}$. One notable departure from Kline and Walters (2016) is that we do not rely on a group fixed effects procedure, which is unnecessary because we have only 8 (rather than 138) randomization strata and can estimate region fixed effects without incidental parameters problems. Like Kline and Walters (2016), however, we estimate the probit model's parameters via simulated maximum likelihood using the GHK simulator (Geweke, 1989; Keane, 1994; Hajivassiliou and McFadden, 1998). We use the first-step choice model parameters to form control function estimates $\{\hat{\lambda}_h(W_i, Z_i, \mathbf{C}_i, D_i), \hat{\lambda}_c(W_i, Z_i, \mathbf{C}_i, D_i)\}$, evaluated at each applicant's own offer cohort, which we use in a second-step regression of equation 12 with treatment-specific intercepts and selection coefficients. To avoid overfitting, first- and second-step models include a subset of available covariates, and only a subset of these enter $\mu_d(W_i)$ with treatment-specific slopes.[3] Outcomes defined only for homebuyers, such as mortgage size, are missing for renters; for these outcomes we estimate the second step on homebuyers only and report subLATEs for inframarginal takers only. To form subLATEs, we evaluate the first-step choice model with the cohort indicators set to zero, so that the complier groups, their weights, and the conditional means of the taste shocks are those of a first-round offer. We estimate standard errors using the nonparametric block bootstrap, re-estimating both steps in each replication.

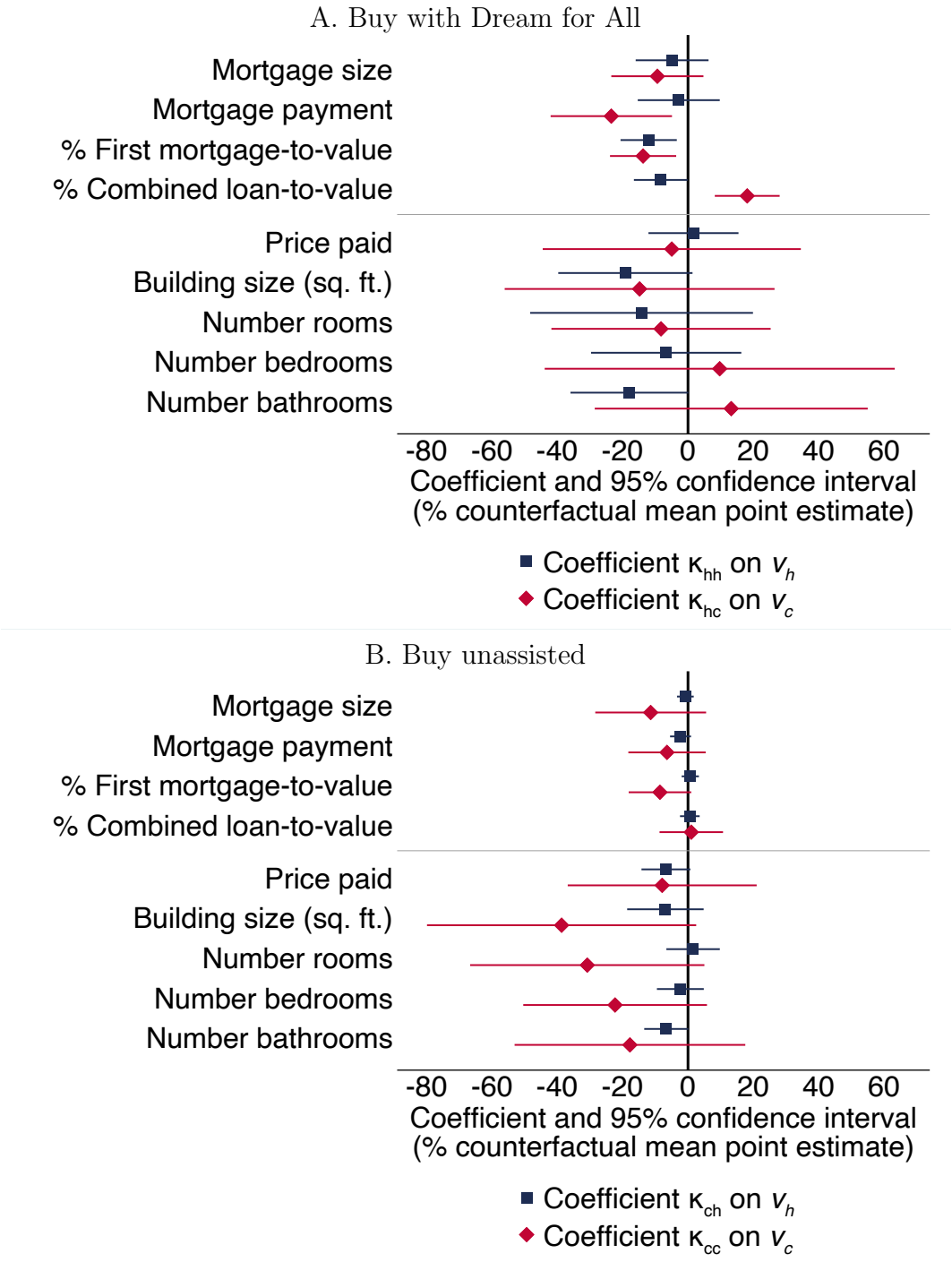
[3]The subset comprises region indicators, mortgage pre-approval quintile indicators, income quintile indicators, household size indicators, an indicator for being female, a Hispanic or Latino indicator, a Black or African American indicator, age bin indicators, and credit score quintile indicators. Region, pre-approval, income, and household size indicators enter $\mu_d(W_i)$ with treatment-specific slopes; the remaining covariates enter with common slopes.

Figure D.1 shows estimated selection coefficients for purchase terms outcomes (as shown in Table 1 in the main text). Panel A shows selection coefficients for the "Buy with Dream for All" outcomes model, and Panel B shows selection coefficients for the "Buy unassisted" outcomes model. We scale coefficients by the selection model's estimate of inframarginal takers' mean counterfactual outcome under buying unassisted, $E\left[Y_i(c) \mid D_i(1) = h, D_i(0) = c\right]$, shown in column (2) of Table D.1, to make coefficients comparable in size across outcomes; a coefficient of -10 means that a one standard deviation increase in the taste shock lowers the outcome by 10% of its counterfactual mean. For interpretation, note that statistically and economically insignificant selection coefficients in the Buy with Dream for All outcomes model are consistent with the RIO assumption in Angrist, Santos and Tecchio (2026): under equation 11, the difference between inframarginal takers' and marginal buyers' mean potential outcomes under Dream for All is κ_{hh} times the difference in the two groups' mean v_{ih} plus κ_{hc} times the difference in their mean v_{ic}, so $\kappa_{hh} = \kappa_{hc} = 0$ implies RIO.

Selection coefficients in the Buy with Dream for All outcomes model (Panel A) are generally statistically indistinguishable from zero for housing consumption outcomes. For financing outcomes, we see some evidence of selection: applicants with stronger unobserved taste for buying unassisted have lower monthly payments, lower first mortgage-to-value ratios, and higher combined loan-to-value ratios, i.e., larger Dream for All loans, when they buy with Dream for All. Wald tests of $\kappa_{hh} = \kappa_{hc} = 0$ based on the bootstrap covariance matrix reject at the 5% level for mortgage size, mortgage payment, and both loan-to-value ratios, and do not reject for any housing consumption outcome. This is the pattern under which the selection model and AST estimates can differ, since RIO assigns marginal buyers and inframarginal takers the same mean financing outcomes under Dream for All. Corresponding estimates of selection coefficients are statistically indistinguishable from zero in the Buy unassisted outcomes model (Panel B) across both housing consumption and financing outcomes.

Table D.1 shows the resulting selection model estimates of the effects of Dream for All utilization on purchase terms alongside the nonparametric estimates from the main text. The selection model's estimates of mean outcomes when buying unassisted in column (2) are within a few percent of the nonparametric estimates in column (1). Panel A shows selection model estimates of mortgage size and payment reductions are two to three times larger than AST nonparametric estimates, as are effects on loan-to-value ratios, although they are much less precisely estimated. The effects are qualitatively similar across methods, however. The same pattern holds across housing consumption outcomes (Panel B).

Figure D.1: Selection coefficients in outcome models



Notes: Markers show the selection coefficients κ_{dh} (squares) and κ_{dc} (diamonds) in equation 11, the coefficients on the taste shocks v_{ih} and v_{ic} in the potential outcome model for buying with Dream for All (Panel A) and for buying unassisted (Panel B), estimated from the second-step regression of equation 12 on homebuyers. Each coefficient is divided by the selection model's estimate of inframarginal takers' mean outcome when buying unassisted (column (2) of Table D.1) and multiplied by 100, so a value of −10 means that a one standard deviation increase in the taste shock lowers the outcome by 10% of its counterfactual mean; for the two loan-to-value ratios this is a percent of the counterfactual ratio, not a change in percentage points. Whiskers are 95% confidence intervals from the nonparametric block bootstrap.

Table D.1: Dream for All utilization effects on purchase terms and property characteristics: nonparametric and selection model estimates

	Counterfactual: buying unassisted		Dream for All effect			
			AST		Selection model	
	Nonparametric	Selection model	Estimate	95% CI	Estimate	95% CI
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Financing						
Mortgage size	555,541	538,894	−74,210	(−102,686, −45,734)	−100,190	(−158,023, −42,358)
Mortgage payment	4,116	3,971	−342	(−559, −125)	−975	(−1,546, −405)
% First mortgage-to-value	89.7	91.1	−9.3	(−10.0, −8.6)	−26.3	(−33.1, −19.6)
% Combined loan-to-value	89.7	90.3	9.9	(9.2, 10.6)	20.9	(10.1, 31.7)
Panel B. Housing consumption						
Price paid	639,882	601,541	−18,181	(−38,563, 2,201)	−14,264	(−235,473, 206,945)
Building size (square feet)	1,450.8	1,489.0	−79.9	(−136.2, −23.7)	−345.9	(−947.1, 255.4)
Number rooms	6.1	6.3	−0.3	(−0.5, 0.0)	−1.1	(−2.9, 0.8)
Number bedrooms	2.9	3.0	−0.1	(−0.2, −0.1)	0.1	(−1.3, 1.6)
Number bathrooms	2.0	2.0	0.0	(−0.1, 0.1)	0.2	(−0.6, 0.9)

Notes: Each row reports estimates of the effects of Dream for All utilization on home purchase terms and property characteristics outcomes for inframarginal takers, applicants who buy unassisted without a Dream for All offer and with Dream for All when offered. Columns (1) and (2) report their average outcome when buying unassisted: column (1) is the nonparametric IV estimate from system 4, as in Table 1; column (2) is the selection model's estimate. Columns (3)–(4) report the effect of buying with Dream for All relative to buying unassisted under the representation-in-observables assumption with 95% confidence intervals from the nonparametric block bootstrap, reproduced from Table 1. Columns (5)–(6) report the selection model's estimate of the same effect, the subLATE of equation 11 averaged over Dream for All buyers, with 95% confidence intervals from the nonparametric block bootstrap re-estimating both steps of the model in each replication; column (2) plus column (5) is the model's estimate of inframarginal takers' average outcome when buying with Dream for All. The selection model sample adds waitlisted applicants who later received an offer to the main analysis sample. Mortgage size and monthly payment (principal, interest, taxes, and insurance) are from credit records; price paid and property characteristics are from property records. First mortgage-to-value and combined loan-to-value ratios are relative to the purchase price; the combined ratio includes the Dream for All loan.